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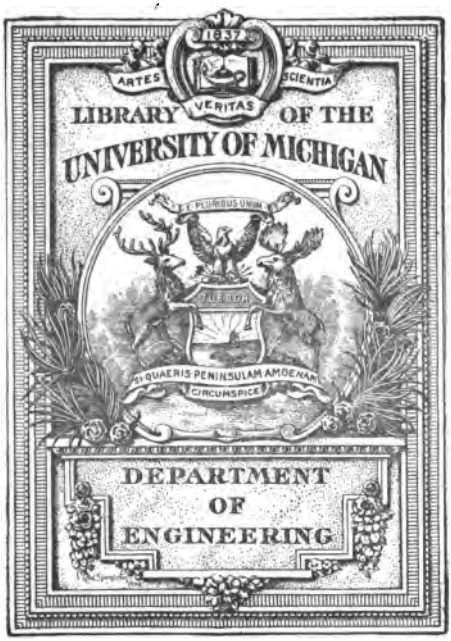
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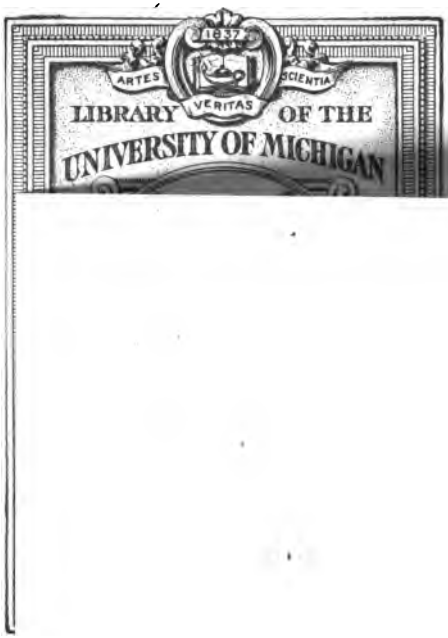
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SYLLABUS

OF

LECTURES AND NOTES

ON THE ELEMENTS OF

MACHINE DESIGN

BY

WALTER RAUTENSTRAUCH

Assistant Professor of Machine Design

Sibley College, Cornell University

NEW YORK

D. VAN NOSTRAND COMPANY

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1906

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PREFACE.

THESE notes are collected in this form in order that the students attending the author's lectures during the Junior year in Sibley College may have a ready reference to all formulas used. Blank pages are inserted in order that frequent notes may be made. The author acknowledges indebtedness to all who have preceded him in the field and from whose works he has largely drawn. Especially from Barr's "Notes on the Design of Machine Elements" and Smith's "Machine Design" was much taken. Mr. T. J. Rodhouse kindly loaned his notes on Mechanics from which Chapters XIII and XV were arranged. Professor W. N. Barnard kindly assisted in various parts of the work. To all of these and to others who have at any time assisted in any of the work the author wishes to express his thanks.

WALTER RAUTENSTRAUCH.

October, 1905.

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SYLLABUS OF LECTURES AND NOTES

ON THE

ELEMENTS OF MACHINE DESIGN.

CHAPTER I.

MATERIALS OF MACHINES.

THE materials for machine members are selected according to their

- (*a*) Properties
  - Resilience
  - Strength
  - Coefficient of Friction
- (*b*) Adaptability
  - for Casting
  - for Forging

The stresses at which materials should be worked will depend on

- (*a*) Nature of Stresses to be resisted
  - Constant (dead load)
  - Repeated (live load)
  - Shocks
  - Unknown Stresses
- (*b*) Nature of Material
  - Homogeneity
  - Internal Stresses
  - Flaws

The materials commonly used for machines and their properties are given in the following table from "Materials of Machines," by A. W. Smith:

Material.	Tensile Strength.	Compressive Strength.	Resilience.
High-carbon steel	Very high	Very high	Medium
Mild steel { Bessemer, open-hearth }	High	High	High
Wrought iron	Medium	Medium	High
Cast iron	Low	Very high	Low
Malleable iron	Medium	.....	High
Steel casting	High	High	High
Brass or Bronze	Low	.....	Medium
White metal	Very low		

CHAPTER II.

SIMPLE BEAMS.

1. Analysis of external forces.
2. Analysis of internal stresses.
3. Shape of material to resist internal stresses.
4. Beams of uniform strength.

NOTATION.

P = concentrated load in pounds;
w = uniform load per inch;
W = uniform total load;
l = length in inches;
M = bending moment, inch-pounds;
E = modulus of elasticity;
I = rectangular moment of inertia;
I_p = polar moment of inertia;
δ = deflection in inches;
k = radius of gyration, inches;
A = area, square inches;
h = height, inches;
b = breadth, inches;
f_t = maximum allowable fiber stress per square inch tension;
f_c = maximum allowable fiber stress per square inch compression;
f_s = maximum allowable fiber stress per square inch shear;
e_t = distance of neutral axis from most strained fiber in tension;
e_c = distance of neutral axis from most strained fiber in compression;
J = shear;
$Z = \frac{I}{e}$ = section modulus.

FORMULÆ.

The following formulæ have been collected for convenient reference in the solution of problems:

1. Fig. 1 Maximum moment $= Pl$;

" deflection $= \frac{1}{3}\frac{Pl^3}{EI}$;

" shear $= P$.

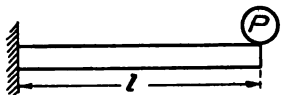


FIG. 1

FIG. 2

2. Fig. 2 Maximum moment $= \frac{1}{2}Wl$;

" deflection $= \frac{1}{8}\frac{Wl^3}{EI}$;

" shear $= W$.

3. Fig. 3 Maximum moment $= \frac{1}{4}Pl$;

" deflection $= \frac{1}{48}\frac{Pl^3}{EI}$;

" shear $= \frac{1}{2}P$.

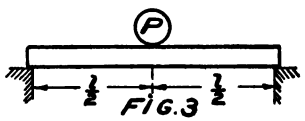


FIG. 3

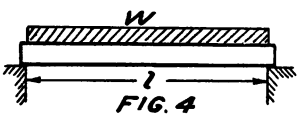


FIG. 4

4. Fig. 4 Maximum moment $= \frac{1}{8}Wl$;

" deflection $= \frac{5}{384}\frac{Wl^3}{EI}$;

" shear $= \frac{1}{2}W$.

5. Fig. 5 Maximum moment $= P_2 l_2$ or $P_1 l_1$;

" shear $= P_2$ if $l_2 < l_1$;

" deflection occurs at a distance

$$= \sqrt{\frac{l_1}{3}(2l - l_1)}$$

from the left-hand support and is equal to

$$\frac{1}{9}\frac{P}{EI}(l_1^3 - 3l_1^2 l + 2l_1 l^2)\sqrt{\frac{l_1}{3}(2l - l_1)}.$$

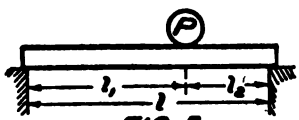


FIG. 5

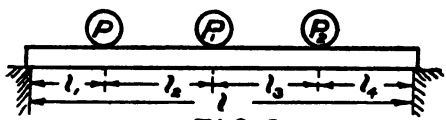


FIG. 6

6. Fig. 6 Maximum moment occurs where shear is zero. Thus if $P_1 + P_2 > P_B < P_2$, the maximum moment is under P_1 and is $P_B(l_3 + l_4) - P_2 l_3$;
Maximum shear $= P_A$ or P_B.

7. Fig. 7 Maximum moment $= l\left(\frac{P}{4} + \frac{W}{8}\right)$;

" deflection $= \frac{l^3}{EI}\left(\frac{P}{48} + \frac{5W}{384}\right)$;

" shear $= (\frac{1}{2}P + W)$.

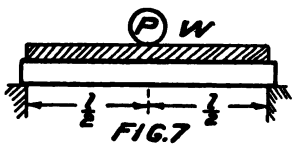


FIG. 7

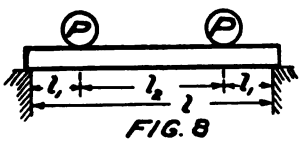


FIG. 8

8. Fig. 8 Maximum moment $= Pl$;

" deflection $= \frac{Pl_1}{24EI}(3l^2 - 4l_1^2)$;

" shear $= P$.

9. Fig. 9 Maximum moment $= \frac{1}{8}Wl + Pl$;

" deflection $= \frac{1}{24EI}[Pl_1(3l^2 - 4l_1^2) + \frac{5}{16}Wl^3]$;

" shear $= \frac{1}{2}W + P$.

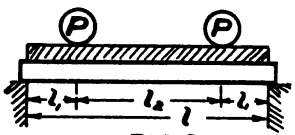


FIG. 9

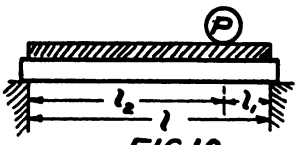


FIG. 10

10. Fig. 10 Maximum moment $= l(P + \frac{1}{2}W)$;

" deflection $= \frac{l^3}{EI}(\frac{1}{3}P + \frac{1}{8}W)$;

" shear $= W + P$.

11. Fig. 11 Maximum moment is under P or between P and the middle of the span depending on whether

$$\frac{P}{W} > \text{ or } < \frac{l_2 - \frac{1}{2}l}{l_1}.$$

For the first case the

$$\text{Maximum moment} = \frac{Pl_1l_2}{l} + \frac{Wl_2}{2} - \frac{1}{2}\frac{Wl_2^2}{l}.$$

For the second case the

$$\text{Maximum moment} = \left[\frac{Pl_1}{l} + \frac{1}{2}W\right]\left[\frac{Pl_1}{W} + \frac{1}{2}l\right] - \frac{1}{2}\frac{W}{l}\left[\frac{Pl_1}{W} + \frac{1}{2}l\right]^2;$$

Maximum shear $= P$ when $l_1 < l_2$.

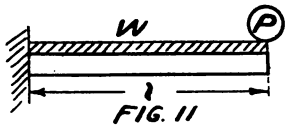


FIG. 11

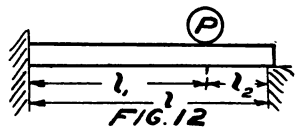


FIG. 12

12. Fig. 12 Maximum moment $= \frac{Pl_1^2l_2}{2l^3}(3l - l_1)$;

$$\text{deflection} = \frac{Pl_1^2}{6EI}(l - l_1)\sqrt{\frac{l - l_1}{3l - l_1}};$$

shear $= P_A$ or P_B.

13. Fig. 13 Maximum moment $= \frac{1}{8}Wl$;

" deflection $= 0.0054\frac{Wl^3}{EI}$;

" shear $= \frac{1}{2}W = P_A$.

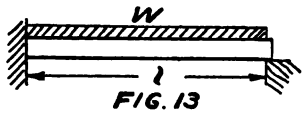


FIG. 13

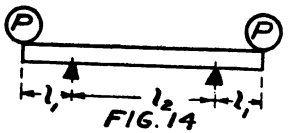


FIG. 14

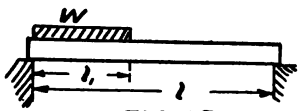
14. Fig. 14 Maximum moment $= Pl_1$;

" deflection $= \frac{1}{3}\frac{Pl^3}{EI}$;

" shear $= P$.

15. Fig. 15 Maximum moment $= \frac{1}{2}Wl\left(1-\frac{l_1}{2l}\right)^2$;

" shear $= \frac{W}{l}\left(l-\frac{l_1}{2}\right)$.



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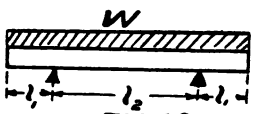


FIG. 16

16. Fig. 16 When $l_2 > l_1\sqrt{8}$, then

Maximum moment $= \frac{W}{2l}[\frac{1}{4}l_2^2 - l_1^2]$;

When $l_2 < l_1\sqrt{8}$, then

Maximum moment $= \frac{Wl_1^2}{2l}$.

When $l_1 > \frac{1}{4}l$, then

Maximum shear $= wl_1$;

When $l_1 < \frac{1}{4}l$, then

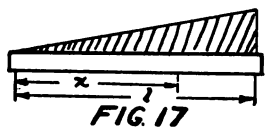
Maximum shear $= \frac{1}{2}W - wl_1$.

17. Fig. 17 Maximum moment $=\frac{1}{9\sqrt{3}}\gamma b l^3$ and occurs at $x=\frac{l}{\sqrt{3}}$,

where γ = weight per cubic unit,

b = constant breadth;

Maximum shear $=\frac{1}{6}\gamma b(l^2-3x^2)$.



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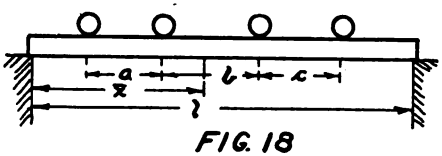

FIG. 18

18. Fig. 18 The maximum moment *under any wheel* occurs when that wheel and the resultant pressure of all the wheels *on the span* are equidistant from the center of the span. The maximum shear is found by computing the reaction in each case when the wheel is just passing off and taking the largest of these.

19. Fig. 19 Maximum moment $=\frac{1}{8}Pl=M_A=M_B=M_C$;

" deflection $=\frac{1}{192}\frac{Pl^3}{EI}$;

" shear $=\frac{1}{2}P$.

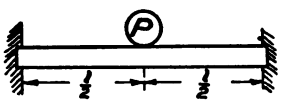

FIG. 19

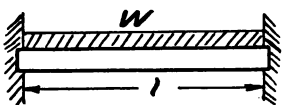

FIG. 20

20. Fig. 20 Maximum moment $=\frac{1}{8}Wl=M_A=M_B=M_C$;

" deflection $=\frac{1}{384}\frac{Wl^3}{EI}$;

" shear $=\frac{1}{2}W$.

BEAMS OF UNIFORM STRENGTH FOR MATERIALS OF EQUAL STRENGTH IN TENSION AND COMPRESSION.

The moment in any case may be found from the preceding formulæ. The outline of the beam may be found from the accompanying equations, where

h = maximum height;
b = maximum breadth;
x = distance from support;
h_1 = height at x;
b_1 = breadth at x.

1. Fig. 21 h = constant;

$$b_1 = \frac{b(l-x)}{l}.$$

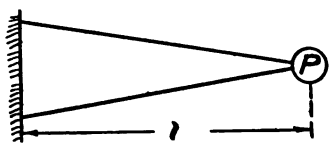

FIG. 21

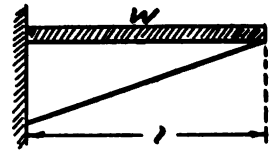

FIG. 22

2. Fig. 22 b = constant;

$$h_1 = \frac{h(l-x)}{l}.$$

3. Fig. 23 h = constant;

$$b_1 = \frac{2bx}{l}.$$

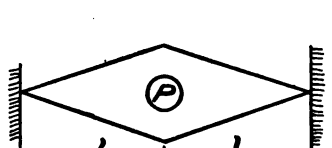

FIG. 23

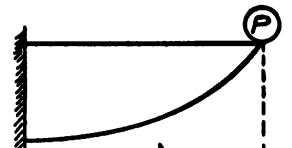

FIG. 24

4. Fig. 24 b = constant;

$$h_1 = h\sqrt{\frac{l-x}{l}}.$$

5. Fig. 25 h=constant;

$$b_1 = b\sqrt{\frac{2(l-x)}{l}}.$$

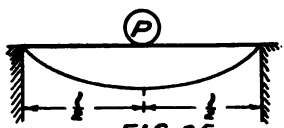

FIG. 25

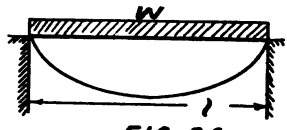

FIG. 26

6. Fig. 26 b=constant;

$$h_1 = \frac{h}{l}\sqrt{lx - x^2}.$$

CHAPTER III.

BENDING AND DIRECT STRESS.

I.

Combined Bending and Direct Stress.

P = load;
a = eccentricity of load.
Other notation as before.

Fiber stress due to direct stress $= f' = \pm \frac{P}{A}$.

Maximum fiber stress due to bending $= f'' = \pm \frac{Pae}{I}$.

Maximum fiber stress due to both causes =

$$f = f' + f'' = \pm \frac{P}{A} \pm \frac{Pae}{I}.$$

Whether or not the factors f' and f'' are + or − must be seen from inspection.

II.

Graphic Solution for Center of Gravity of an Irregular Figure.

It will at once appear from the above equation that f cannot be found unless A and I and center of gravity, upon which e depends, are known. These values may be readily found for any

regular section. For irregular sections as frequently found in machine frames, these values are found graphically as follows.

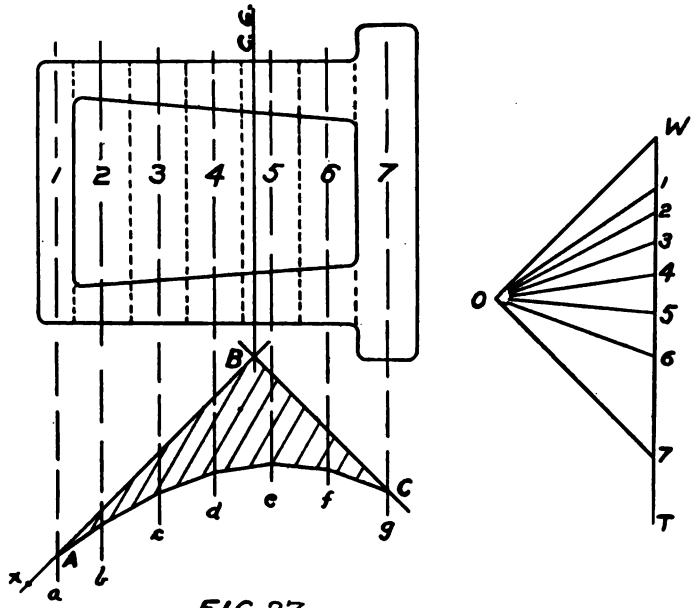

FIG. 27

A. Lay out the section to convenient scale. The section divides itself into four regular parts, so that the area of the entire section will be the sum of these four partial areas. If the section happens to be more complex, a greater number of parts must be chosen or a planimeter may be conveniently used.

C. G. Divide the section into a convenient number of strips (1, 2, 3, 4, 5, 6, 7). Through the center of gravity of each strip, which, if the strips are small, may be taken half-way between the sides, draw a number of parallel lines (*a*, *b*, *c*, *d*, *e*, *f*, *g*) as shown. Next lay off a line *WT*, parallel to the strips, and on this line mark off the distances 1, 2, 3, 4, 5, 6, 7, respectively proportional to the area of the strips.

From the extremities of these divisions draw lines at an angle of 45 degrees, intersecting at *O*. Draw the rays to the extremities of each division as shown.

Begin at some convenient point x and draw a line parallel to OW and in extent as shown. Where this line intersects (a) draw a line parallel to 01 intersecting (b). From this intersection draw a line parallel to 02, and so on until the entire diagram is drawn, the line drawn parallel to 07 being extended until it intersects the line drawn parallel to OW. At this point of intersection draw a line parallel to the lines a, b, c, etc., and it will pass through the center of gravity of the section. If the location of the center of gravity of an unsymmetrical figure is to be found, this process must be repeated by taking strips at right angles, the intersection of the two gravity lines locating the center. A gravity axis is usually all that is required.

The moment of inertia is sometimes found graphically by multiplying the shaded area ABC by the area of the section and introducing the proper scale factor. This is, however, not recommended since the moment of inertia may easily be found by the usual methods given in works on Mechanics.

CHAPTER IV.

TORSION AND BENDING.

I.

Simple Torsion.

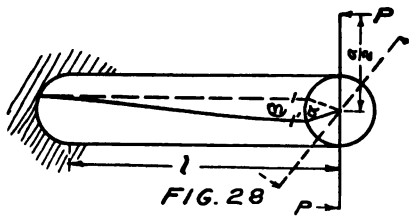

FIG. 28

With the notation indicated in the figure the following formulæ may be derived:

$$Pa = \frac{f_s I_P}{e}.$$

$$Pa = 63024 \frac{\text{H.P.}}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

H.P. = horse-power;

N = revolutions per minute.

$$d = \sqrt[3]{\frac{5.1Pa}{f_s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$d = 68.5 \sqrt[3]{\frac{\text{H.P.}}{Nf_s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$f_s = \frac{e\alpha E_s}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

α = angle of torsion in π measure;
E_s = modulus of shearing.

$$f_s = \frac{e\theta E_s}{57.3l} \quad \ldots \ldots \ldots \ldots \quad (5)$$

θ = angle of torsion in degrees.

$$d = \sqrt[4]{584\frac{Pal}{\theta E_s}} = 4.9\sqrt[4]{\frac{Pal}{\theta E_s}} \quad \ldots \ldots \quad (6)$$

$$d = 77.66\sqrt[4]{\frac{\text{H.P.}l}{N\theta E_s}} \quad \ldots \ldots \quad (7)$$

II.

Combined Torsion and Bending.

The formulæ for combined torsion and bending may be derived as follows:

Let a small triangular plate as shown, of small thickness, be removed from the surface of the shaft. The stresses on such a plate are as represented in the figure. The stresses (f) are induced by the torsion, (f_1) is induced by the bending, and (f_2) is the resulting tensile stress occurring on the surface AC, so taken that no shearing stress appears; in other words, it is normal to the resultant of f and f and f_1. In accord with this the value of θ may be found as follows:

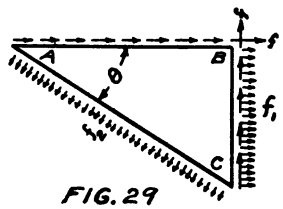

FIG. 29

Summing the forces in the direction AC we have

$$f(AB\cos\theta) + f_1(BC\cos\theta) = f(BC\sin\theta);$$

whence

$$O = f(\cos^2\theta - \sin^2\theta) + f_1\sin\theta\cos\theta,$$

and

$$\tan 2\theta = -\frac{2f}{f_1}.$$

The value of the stress on the face AC is found as follows: Summing the forces normal to the face AC, we have

$$f_2 AC = f_1 BC \sin\theta - fAB \sin\theta + fBC \cos\theta;$$
$$f_2 = f_1 \sin^2\theta + 2f \sin\theta \cos\theta;$$
$$f_2 = \tfrac{1}{2} f_1 (1 - \cos 2\theta) + f \sin 2\theta.$$

Substituting for $\cos 2\theta$ and $\sin 2\theta$ in terms of $\tan 2\theta$, we have

$$f_2 = \tfrac{1}{2} f_1 + \sqrt{f^2 + \tfrac{1}{4} f_1^2}$$
$$= \tfrac{1}{2}(f_1 + \sqrt{f_1^2 + 4f^2}).$$

Letting M_1 represent the bending moment producing f_1, and T represent the torsional moment producing f and f, we may combine these into an equivalent moment M_2, producing the stress f_2; thus we have

$$f = \frac{T}{\frac{\pi}{16} d^3};$$

$$f_1 = \frac{M_1}{\frac{\pi}{32} d^3};$$

$$f_2 = \frac{M_2}{\frac{\pi}{32} d^3};$$

whence

$$\frac{M_2}{\frac{\pi}{32} d^3} = \frac{1}{2} \frac{M_1}{\frac{\pi}{32} d^3} + \frac{1}{2} \sqrt{\left(\frac{M_1}{\frac{\pi}{32} d^3}\right)^2 + 4\left(\frac{T}{\frac{\pi}{16} d^3}\right)^2},$$

or

$$M_2 = \tfrac{1}{2} M_1 + \tfrac{1}{2} \sqrt{M_1^2 + T^2}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For a solid circular shaft the above may be put into the convenient form

$$d = \sqrt[3]{\frac{16}{\pi f_t}(M + \sqrt{M^2 + T^2})}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For hollow shafts such as are used aboard ship, if we let

d = outside diameter,
d' = inside diameter,
$d' = nd$,

we may write for simple torsion

$$d = \sqrt[3]{\frac{5.1Pa}{(1-n^4)f_s}}, \quad \ldots \quad (10)$$

$$d = 68.5\sqrt[3]{\frac{\text{H.P.}}{(1-n^4)Nf_s}}, \quad \ldots \quad (11)$$

and for combined torsion and bending

$$d = \sqrt[3]{\frac{16}{\pi(1-n^4)f_s}(M+\sqrt{M^2+T^2})}. \quad \ldots \quad (12)$$

III.

Graphic Solution for Combined Torsion and Bending.

When cases arise in which there are several forces acting on a shaft, producing bending in different planes and torsion, the solution is more readily carried out graphically.

As a simple case consider a shaft on which are acting forces at an angle α and which produce bending moments and a torsional moment as shown in the figure.

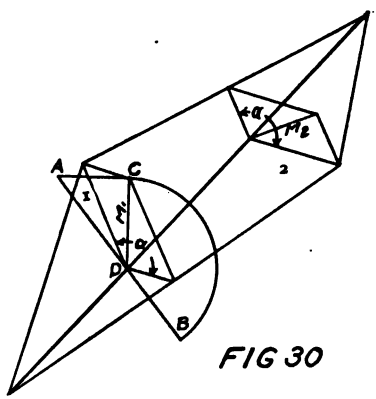

FIG 30

Draw the center line of the shaft at any convenient angle (in this case it is taken, 45°), and at the several points where the forces act draw in the maximum moments 1 and 2 to scale. These forces act normal to the shaft.

Combine the ordinates of the moment diagrams at the points where the forces act and obtain the resultants M_1 and M_2. These are the resultant bending moments at these points, one of which is a maximum. Letting M_1 be found to be the maximum moment, it will be necessary to combine it with the torsional moment in the shaft to get the equivalent torsional moment.

If the equivalent torsional moment is T_2, we may write

$$T_2 = M_1 + \sqrt{M_1^2 + T^2}.$$

Graphically this may be solved thus: Lay off AC equal to T to scale and at right angles to M_1 as shown. Join AD and extend the line to B, a distance equal to DC. AB then represents to scale the equivalent torsional moment in the shaft.

CHAPTER V.

FORMS DICTATED BY INTERNAL STRESSES.

In the foregoing chapters have been derived the formulæ relating the external forces on a machine part to the internal stresses they produce. Let us now examine these formulæ more closely and see if the factors determining the internal stresses do not indicate certain forms more suited than others. Take first the simple formula for tension, $f_t = \frac{P}{A}$. This gives no definite idea of form—area is all that is required, and one may use a circular or rectangular section as desired, being careful, of course, to see that the tension is uniform over the entire section. Economy of manufacture or appearance may sometimes decide the form to use. For compression the same formula may be used for short members and the section decided as above. For long columns we have $f = \frac{P'A}{1 + \frac{f}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2}$. Among other things this formula shows that the stress varies inversely as the radius of gyration of the section. A section with the material distributed well out from the axis gives the desired result; therefore, if bending is liable in one plane only, which can be determined from the end conditions, an I section, or rectangular section if the I is too expensive, may be used. A hollow circular section will give the best results for liability of bending in any plane, or a hollow square or box section, if it can be produced cheaper and

more conveniently used. Take the stress due to flexure, $f = \frac{Me}{I}$. This formula indicates that the stress varies inversely as the moment of inertia of the section. This suggests the I form of section if flexure occurs in one plane only. If it is liable to occur in a plane at right angles also, the box form would probably meet these requirements better than any other form. The same is also true of torsion. In machine frames, as a rule, the stresses acting are a combination of these simple stresses; that is, the frame may be subject to flexure, torsion, compression, at the same or different times. It will be required, then, to have a form of section which will take up any combination of these stresses. It seems, from what has been said already, that a hollow section is the best suited, and of the hollow sections the box form seems to be the most advantageous. Professor A. W. Smith, in his "Machine Design," says of the box form:

I. "Its appearance is far finer, giving an idea of completeness that is always wanting in the ribbed frames."

II. "The faces of a box frame are always available for the attachment of auxiliary parts without interfering with the perfection of the design."

III. "The strength can always be increased by decreasing the size of the core, without changing the external appearance of the frame, and therefore without any work whatever on the pattern itself."

Besides a thorough knowledge of the principles of mechanics one must have the faculty to recognize other factors entering into the design of machinery in order to become a successful designer. Some of the things that may influence the design are:

1. Cost of patterns or core-work.
2. Cost of machine-work.
3. Interchangeability.
4. Effect of temperature stresses.
5. Special adaptation.

It may be well to examine the stresses occurring in some typical machine.

The design of the punch may well serve to illustrate the application of the foregoing principles.

Fig. 31 will represent a good form of punch frame, and the analysis of the stresses will be confined to sections (*a*), (*b*), and (*c*).

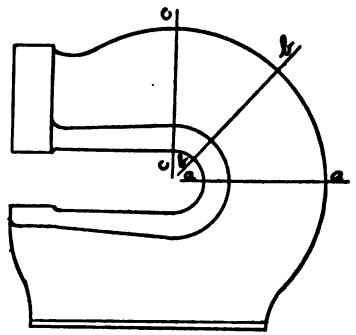

FIG 31

At (*a*) the stresses will be a combination of direct tension equal to P and stresses due to the bending moment $P(l+x)$. The solution for a section of this kind must be made by trial. We will assume a section as shown in Fig. 27. It will be noted that all the corners are well rounded and that no abrupt changes in section occur, which is what is required to reduce temperature stresses to a minimum. It will also be noted that since this must needs be made of cast iron, which has a higher allowable stress in compression than in tension, the material is more concentrated on the tension side in order to bring the gravity axis nearer the tension side, so that e_t becomes smaller than e_c, upon which factors the respective stresses directly depend. The ratio of e_t to e_c cannot be made equal to the ratio of maximum allowable stresses in tension and compression respectively, because the machine would look out of proportion or the section may be a bad one for casting. The first thing to be done, then, is to assume some dimensions for a section such as found in Fig. 27, and, as given in the preceding chapter, find the moment of inertia, e_t, and the area. The maximum tensile stress will be

$$f_t = \frac{P}{A} + \frac{Ple_t}{I}.$$

Suppose dimensions are assumed for a section as given above and that for a given force P it is found that $f_t = 3000$ lbs. per square inch. This is somewhat too large a value to use for the

load being suddenly applied, and, there being also temperature stresses which cannot be computed, the actual stresses in the frame are very uncertain. It is therefore advisable to use a lower stress, which has been found in practice to give good results. Let it be assumed that the maximum stress in tension shall not exceed 2200 lbs. per square inch, how is the design to be altered? It will be noted that if all the dimensions of the section are multiplied by any given factor, I will vary as the fourth power of the factor, A as the square, and e_t directly. Therefore we may select by trial some factor which, when used in the equation for f_t, will give a value near 2200 lbs. per square inch. The dimensions of the trial section when multiplied by this factor will then give the proper dimensions to the section. If a punch frame has been designed for a given load and patterns for the same are made, the same pattern may be used for a punch to use a greater force if a smaller core is used, thus giving a larger section.

At B the force is not normal to the plane of the section and we therefore have a shear over the section as well as a direct tension and stresses due to flexure. The shear is usually not a deciding factor. The tension stress is usually greater than any and decides the proportions of the section. Let the section B make an angle α with the horizontal and it will appear that the direct tension normal to the section will be $P \cos \alpha$, and the tension due to flexure will be $\frac{Pl_1e_t}{I}$, whereupon

$$f_t = \frac{P \cos \alpha}{A} + \frac{Pl_1e_t}{I}.$$

The section may be assumed of the same proportions as the horizontal section, and the dimensions may be found as in the preceding.

When $\alpha = 90°$ the same analysis will apply to a vertical section. These proportions may again have to be altered to give the machine a good outline or for the attachment of parts.

The above analysis may be applied equally well to any kind of machine of the same form, such as slotting-machines, etc. When the above frame is used for a shear there will also exist a torsional stress as well as the others mentioned. A somewhat lower fiber stress, according to the designer's judgment, will account for this.

CHAPTER VI.

STRESSES IN GEAR-WHEELS.

I.

STRESSES IN SPUR-GEAR TEETH.

It is usual to consider the load as coming entirely on one tooth, and at the maximum obliquity of action when the load acts on the end of the tooth. Also, when the width of face does not exceed about three times the circular pitch, it may be assumed that the load is evenly distributed along the whole face of the tooth. According to the above, and as illustrated in the figure, the moment acting on the tooth is $P'X'$ and the weakest section is at aa', from which we have

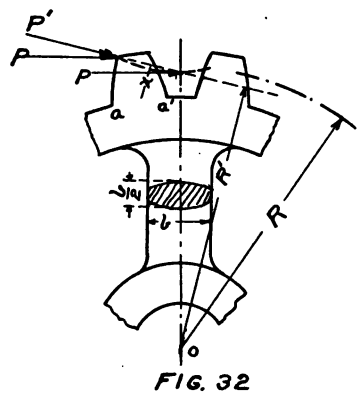

FIG. 32

$$P'X' = PX = \frac{fI}{e},$$

I and e being taken at the section aa'.

This distance aa' can be seen to depend on the circular pitch of the teeth and the outline of the tooth as follows: For a generating circle proportional to the pitch circle the distance aa' can be expressed in terms of the circular pitch and number of teeth in the gear.

For a generating circle equal in diameter to the radius of a

15-tooth pinion, and fillets with a radius equal to one-sixth the space between the teeth at the addendum circle, we have the formula of Professor Barr for the load per tooth,

$$P = pbf\left(0.106 - \frac{0.678}{n}\right), \quad \ldots\ldots \quad (1)$$

where p = circular pitch in inches;
b = width of gear face, inches;
f = fiber stress in pounds per square inch;
n = number of teeth in gear.

For a generating circle equal in diameter to the radius of a 12-tooth pinion and fillets with a radius equal to the clearance at the bottom of the teeth, we have the formula of Mr. Lewis,

$$P = pbf\left(0.124 - \frac{0.888}{n}\right), \quad \ldots\ldots \quad (2)$$

from which, if D = pitch diameter of gear, we have

$$p = D\left(0.22 - \sqrt{0.049 - \frac{3.57P}{bfD}}\right). \quad \ldots \quad (3)$$

II.

Stresses in Bevel-gear Teeth.

According to Mr. Lewis, it may be assumed that a spur-gear of pitch and diameter equal to the pitch and diameter of the bevel-gear at the larger ends of the teeth is stronger than the bevel-gear in the ratio of D to d, when D is the pitch diameter at the larger end and d the pitch diameter at the smaller end. According to which Professor Barr writes

$$P = pf\left(0.124 - \frac{0.28p}{D}\right)\frac{d}{D}; \quad \ldots\ldots \quad (4)$$

whence

$$P = D\left(0.222 - \sqrt{0.048 - \frac{3.57P}{fd}}\right) \quad \ldots \quad (5)$$

for ordinary length of face.

III.

Stresses in Arms of Gear-wheels, and Proportions of Rim and Nave.

The allowable stress, according to good authority, may be taken as follows:

For cast iron, $f = 8000\left(\frac{600}{600+V}\right)$;

" steel, $f = 20{,}000\left(\frac{600}{600+V}\right)$;

" bronze, $f = 12{,}000\left(\frac{600}{600+V}\right)$;

where V = velocity at pitch line in feet per minute.

An exact analysis of the stresses occurring in the arms of a gear-wheel is not very satisfactory since there are stresses due to shrinkage which cannot be computed. It will be safe to consider the arm as a cantilever, with a length $= R$ and a load at the end $= \frac{P}{N}$, where N = number of arms. (See Fig. 32.) For an elliptical arm as shown, let b and $\frac{b}{2}$ represent the proportions the arm would have if projected to the center; then

$$\frac{P}{N}R = \frac{fI}{e},$$

$\frac{I}{e}$ for ellipse having proportions as shown $= 0.05b^3$; whence

$$\frac{P}{N}R = 0.05fb^3.$$

Because of unknown stress a low fiber stress should be used, so 1000 lbs. per square inch may be taken. The arms at the top are usually tapered to $\frac{3}{4}$ or $\frac{2}{3}$ size.

Other proportions of the gear are established by good practice, for which we may take

Diameter of hub = twice diameter of shaft;

Length of hub = width of rim;

Thickness of rim = thickness of tooth reinforced by a "bead" of same thickness and depth.

Good judgment must always be used, and the designer must always keep in mind how the proportions will cast.

CHAPTER VII.

STRESSES IN SPRINGS.

I.

Stresses in Flat Springs.

NOTATION.

P = load on spring;
L = free length of spring;
f = intensity of stress in outer fiber;
I = moment of inertia of most strained section;
h = dimension of this section in plane of flexure;
b = dimension of this section perpendicular to plane of flexure;
E = modulus of elasticity of material;
δ = deflection of spring;
n = number of strips;
b = breadth of one strip $= \frac{b}{n}$.

From the general theory of beams we have the following formulæ applying to flat springs, as shown in Fig. 33:

$$PL = Afbh^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\delta = B\frac{PL^3}{Ebh^3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where A and B have values for the various types as assigned in the following table:

Type.	Coefficient.	
	A	B
1	$\frac{3}{4}$	$\frac{1}{4}$
2	$\frac{3}{8}$	$\frac{3}{8}$
3	$\frac{1}{6}$	6
4	$\frac{1}{6}$	4

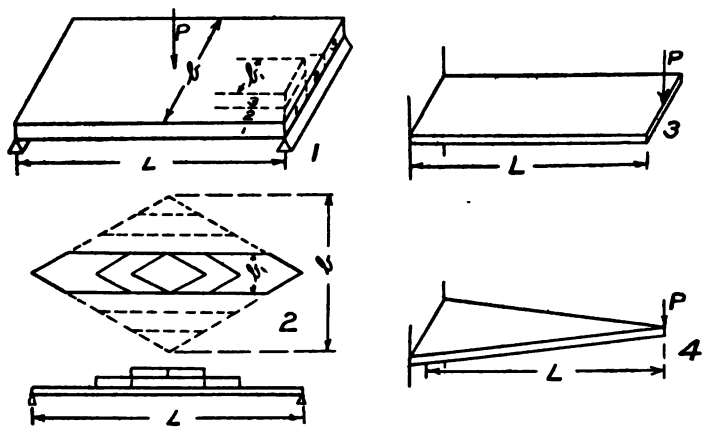

FIG. 33

From the above formulæ, the following formulæ are derived:

$$P=\frac{Enb_1h^3\delta}{BL^3}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$P=\frac{Afnb_1h^2}{L}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$h=\frac{ABfL^2}{E\delta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$f=\frac{E\delta h}{ABL^2}; \quad \ldots\ldots\ldots \quad (6)$$

$$f=\frac{PL}{Anb_1h^2}. \quad \ldots\ldots\ldots \quad (7)$$

In applying the above formulæ to steel springs it is safe to assume

$$E=30{,}000{,}000;$$

$$f=60{,}000+\frac{7500}{h}.$$

In practice springs are sometimes found which are a combination of those forms shown in Figs. 33_1 and 33_2. In this event the number of plates extending the whole length of the spring is usually one-fourth the total number. The deflection then would be equal to the deflection of a spring shown in Fig. 33_2 minus one-fourth the difference between the deflections of types in Figs. 33_1 and 33_2, or

$$\frac{6PL^3}{Enb_1h_3}-\frac{1}{4}\left(\frac{6PL^3}{Enb_1h^3}-4\frac{PL^3}{Enb_1h^3}\right)=\frac{5.5PL^3}{Enb_1h^3},$$

and we have

$$\delta=\frac{11}{32}\left[\frac{PL^3}{Enb_1h^3}\right]. \quad \ldots\ldots\ldots \quad (8)$$

For any other combination the proper coefficient may be found as in the preceding cases. When leaves of different thickness are used, the thickest leaf has the greatest stress. A band in the middle, to secure the plates, reduces the effective length of spring by one-half length of band.

II.

STRESSES IN HELICAL SPRINGS.

Fig. 34 represents a helical spring.

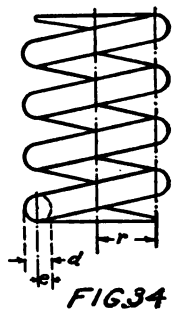

FIG 34

NOTATION.

P = load on spring;
r = radius of spring to *centre of wire;*
e = radius of wire;
d = diameter of wire;
f_s = maximum stress due to torsion
$= 40{,}000 + \frac{15{,}000}{d}$ for mild steel;
E = modulus of shearing
$= 12{,}000{,}000$ for mild steel;
δ = deflection of spring;
n = number of coils in spring;
L = length of wire in spring.

The principal stress induced in a helical spring is torsion, from which we obtain

$$Pr = \frac{f_s I_p}{e}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

since the above may be considered the same as a straight circular rod subject to a turning moment Pr.

It is evident that this equation is true for any section of the spring and is therefore perfectly general. From it may be found, then, the load (P) any spring will carry for an assigned fiber stress when the diameter of the wire and radius of the coil are known. Or, for any given load and allowable stress, what proportions of coil will be needed.

We may solve this equation for the several factors and obtain

$$P=\frac{\pi f_s d^3}{16r}=0.2\frac{f_s d^3}{r}; \quad \ldots\ldots\ldots \quad (10)$$

$$r=\frac{\pi f_s d^3}{16P}=0.2\frac{f_s d^3}{P}; \quad \ldots\ldots\ldots \quad (11)$$

$$d=\sqrt[3]{\frac{16Pr}{\pi f_s}}=1.72\sqrt[3]{\frac{Pr}{f_s}}; \quad \ldots\ldots \quad (12)$$

$$d=\frac{16Pr}{\pi d^3}=5.1\frac{Pr}{d^3}. \quad \ldots\ldots\ldots \quad (13)$$

These equations apply strictly to the strength of the spring.

It is sometimes necessary to know the amount of deflection resulting from a given load, and in order that this may be a given amount or within certain limits it is necessary to know the relation between the dimensions of the spring and the deflection for a given load.

This relation may be arrived at as follows: From the previous analysis it is evident that the spring may be considered as a rod under a torsional moment, so that the deflection of such a rod may be directly applied to the spring. For example, a rod as shown in Fig. 28 is under stress so that the element l is twisted to the position l'.

For any length l it will be seen that

$$\beta l=e\alpha,$$

$$\beta=\frac{e\alpha}{l}.$$

But $\beta=\frac{f_s}{E_s}$; whence

$$f_s=\frac{e\alpha E_s}{l},$$

and substitution in equation (9) gives

$$Pr=\frac{\alpha I_p E s}{l}.$$

In this equation α is the angle of twist, and it will be seen that the distance the load P passes through will be αr.

Consider the same condition in a helical spring, and we have for any portion of the spring as shown that the distance the load passes through (αr) is the deflection of the spring. Letting the deflection αr be expressed by δ, we have

$$Pr = \frac{\delta I_p E_s}{rl},$$

and reducing this equation by substituting for the various factors, we have for $l = 2\pi rn$

$$P = \frac{\delta d^4 E_s}{64 r^3 n}; \quad \ldots\ldots\ldots \quad (14)$$

from which

$$\delta = \frac{64 P r^3 n}{d^4 E_s}; \quad \ldots\ldots\ldots \quad (15)$$

$$n = \frac{\delta d^4 E_s}{64 P r^3}; \quad \ldots\ldots\ldots \quad (16)$$

which equations relate the deflection and proportions of the spring for different loads. These equations may now be combined so as to give a relation between the load and both strength and deflection, whence

$$\delta = \frac{12.57 n r^2 f_s}{E_s d}; \quad \ldots\ldots\ldots \quad (17)$$

$$r = \sqrt{\frac{\delta d E_s}{12.57 f_s}}; \quad \ldots\ldots\ldots \quad (18)$$

$$d = \frac{12.57 n r^2 f_s}{\delta E_s}. \quad \ldots\ldots\ldots \quad (19)$$

CHAPTER VIII.

LONG COLUMNS.

WHEN a short piece of material, one whose ratio of diameter to length is less than about 1 to 6, is subjected to compression, the material will fail by direct crushing of the fibers. However, for a greater ratio of length to diameter, that is, for *long columns*, when compression is applied, failure is mainly due to the buckling of the column. Many formulæ have been derived for such a condition, but the following only will be considered:

I. Euler's formula.
II. Ritter's formula.
III. Rankine-Gordon formula.
IV. Formula for eccentric loading.

I.

EULER'S FORMULA.

Euler's formula is probably the oldest and may be expressed as follows:

$$P' = mEI\frac{\pi^2}{l^2}; \quad \ldots\ldots\ldots \quad (1)$$

where P' = the breaking load;
l = length of column;
m = factor for end conditions as shown in the following figures.

Thus we will have values 1, 4, $\frac{9}{4}$, $\frac{1}{4}$, respectively, for conditions 1, 2, 3, 4.

For the derivation of Euler's formula refer to Church's "Mechanics," Part III, Chap. IV. It will be noted that in the derivation of this formula only the stress due to the buckling action is taken into account. This will give large values when

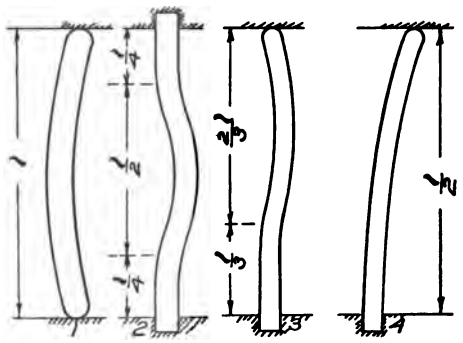

FIG. 35

the column is comparatively short, for it will be evident that for the short column a good portion of the stress will be due to the direct crushing effect. Of course there will be the same amount of direct crushing no matter how long the column, but it is to be understood that for comparatively long columns this is very small compared with the stress due to buckling. For this reason, then, Euler's formula should not be used for columns shorter than a given amount. This may be shown by a diagram (see Fig. 36) as follows:

We may write

$$p=\frac{P'}{A}=mE\frac{I}{A}\frac{\pi^2}{l^2}$$

$$=mE\frac{\pi^2}{\left(\frac{l}{\sqrt{\frac{I}{A}}}\right)^2}$$

$$=mE\frac{\pi^2}{\left(\frac{l}{\rho}\right)^2}.$$

Assume the material shall not be stressed beyond 36,000 lbs. per square inch $= p_{max.}$ and $E = 29{,}400{,}000$, and plotting values of $\frac{l}{\rho}$ to p, we obtain the curve (E) in the accompanying diagram for round-ended columns. Thus it will appear that *under these conditions* Euler's formula should not be applied for values of less than 90.

II.

Ritter's Formula.

Some years ago, Ritter derived the following formula:

$$P' = \frac{fA}{1 + \frac{f}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2}, \quad \ldots\ldots\ (2)$$

where $P' =$ the breaking load;
$f =$ stress at elastic limit.

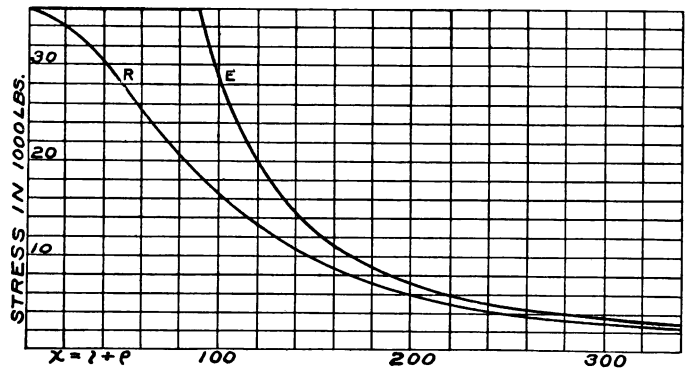

FIG. 36

When the formula is to be solved for any working load P it may be expressed

$$\frac{P'}{k} = P = \frac{\frac{f}{k}A}{1 + \frac{f}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2}, \quad \ldots\ldots\ (3)$$

where k = factor of safety, or

$\frac{f}{k}$ = working stress.

This formula accounts for the direct compressive stress and stress due to buckling, and may be derived as follows: Euler's formula, which considers only the stress due to buckling, may be expressed

$$P = mEI\frac{\pi^2}{l^2},$$

and the stress diagram on the cross-section of maximum stress may be shown as in Fig. 37*a*.

If the diagram for direct compressive stress, Fig. 37*b*, were combined with the diagram for the stress due to buckling, the stress diagram shown in Fig. 37*c* would result.

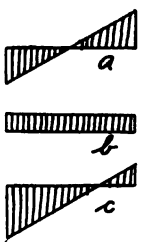

FIG. 37

Let f = maximum stress in Fig. 37*c*;

f' = uniform stress in Fig. 37*b*;

f'' = maximum stress in Fig. 37*a*.

From which it will appear that

$$f''\left(\frac{f}{f-f'}\right) = f.$$

Letting Fig. 37*a* represent the stress diagram obtained by a formula taking into account only the buckling action (Euler's), and Fig. 37*b* represent the stress diagram obtained by a formula taking into account both buckling and crushing (Ritter's), it will be seen that Ritter's formula may be derived from Euler's formula by multiplying the latter by the ratio $\frac{f-f'}{f}$; thus

$$P' = mEI\frac{\pi^2}{l^2}\left(\frac{f-f'}{f}\right)$$

$$= mEI\frac{\pi^2}{l^2}\left(1-\frac{f'}{f}\right)$$

$$= mEI\frac{\pi^2}{l^2}\left(1 - \frac{P'}{Af}\right)$$

$$= \frac{fA}{1 + \frac{fAl^2}{m\pi^2 EI}}$$

$$= \frac{fA}{1 + \frac{f}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2}.$$

If this formula is plotted to the same coordinate as the Euler formula, curve (R) will result, as shown in Fig. 36. Comparing these two curves, E and R, it will be noted that for the smaller values of $\frac{l}{\rho}$ the curves have a greater difference than for the larger values. This of course is to be expected, considering what factors enter into each formula.

Ritter's formula seems to be perfectly general for all values of $\frac{l}{\rho}$, while Euler's formula can only be safely used for the larger values. Ritter's and Euler's formulas are not very far different for the larger values.

III.

Rankine-Gordon Formula.

If β be written for $\frac{f}{\pi^2 E}$, we have

$$P' = \frac{fA}{1 + \frac{1}{m}\beta\left(\frac{l}{\rho}\right)^2}, \quad \ldots\ldots\ldots \quad (4)$$

which is known as the Rankine-Gordon formula. To β are assigned values derived from experiments, hence this formula can only be applied for such cases where experiments have been

performed. This formula is used by a number of steel-mills and may be found in their hand-books.

Professor Barnard has very neatly arranged Ritter's formula in the following manner. Given the dimensions of any section in terms of d, we write

$$\text{Area} = ad^2,$$

$$(\text{Radius of gyration})^2 = \frac{d^2}{b};$$

and also let

$$\frac{f}{m\pi^2 E} = c;$$

whence Ritter's formula will appear as

$$P' = \frac{fad^2}{1 + cb\left(\frac{l^2}{d^2}\right)},$$

whence

$$P' + P'bc\frac{l^2}{d^2} = fad^2;$$

$$fad^4 - P'd^2 = P'bcl^2;$$

$$d^4 - \left(\frac{P'}{fa}\right)d^2 = \left(\frac{P'}{fa}\right)bcl^2;$$

$$d^4 - \left(\frac{P'}{fa}\right)d^2 + \left(\frac{P'}{2fa}\right)^2 = \left(\frac{P'}{2fa}\right)^2 + \left(\frac{P'}{fa}\right)bcl^2;$$

$$d^2 = \frac{P'}{2fa} + \sqrt{\left(\frac{P'}{2fa}\right)^2 + \left(\frac{P'}{fa}\right)bcl^2};$$

$$d = \sqrt{\left(\frac{P'}{2fa}\right)\left(1 + \sqrt{1 + \frac{2bcl^2}{\left(\frac{P'}{2fa}\right)}}\right)}; \quad \ldots\ldots\ (5)$$

or, using P = the working load,

$\frac{f}{k}$ = the working stress,

$$d = \sqrt{\left(\frac{P}{2\frac{f}{k}a}\right)\left(1 + \sqrt{1 + \frac{2bcl^2}{\frac{P}{2\frac{f}{k}a}}}\right)} \quad \ldots\ldots \quad (6)$$

It is convenient to have this formula directly apply to the case of the piston-rod, whence

$$a = 0.785,\ b = 16,\ c = \frac{30{,}000}{1 \times 10 \times 30{,}000{,}000} = \frac{1}{10{,}000}.$$

Since the end conditions are uncertain, m is taken as 1 in solving for c.

The above substitutions give

$$d = \sqrt{\frac{P}{2 \times 0.785\left(\frac{f}{k}\right)}\left(1 + \sqrt{1 + \frac{2 \times 16}{10{,}000} \times \frac{l^2}{\frac{P}{2 \times 0.785\left(\frac{f}{k}\right)}}}\right)}$$

$$= \sqrt{\left(\frac{P}{1.57\frac{f}{k}}\right)\left(1 + \sqrt{1 + \frac{1}{300} \times \frac{l^2}{\frac{P}{1.57\frac{f}{k}}}}\right)} \quad \ldots\ldots \quad (7)$$

For the case of a rectangular-section connecting-rod, where the height is 2.7 times the depth of section, we find

$$a = \frac{1}{2.7};$$

$$b = 12;$$

whereupon for

$$E=30{,}000{,}000,$$
$$f=30{,}000,$$
$$m=1,$$

we find

$$c=10{,}000,$$

and

$$d=\sqrt{\left(\frac{1.35P}{\frac{f}{k}}\right)\left(1+\sqrt{1+\frac{l^2}{420\left(\frac{1.35P}{\frac{f}{k}}\right)}}\right)}, \quad . \quad . \quad . \quad (8)$$

while for

$$E=30{,}000{,}000,$$
$$f=36{,}000,$$
$$m=1,$$

we find

$$c=8330,$$

and

$$d=\sqrt{\left(\frac{1.35P}{\frac{f}{k}}\right)\left(1+\sqrt{1+\frac{l^2}{350\left(\frac{1.35P}{\frac{f}{k}}\right)}}\right)}. \quad . \quad . \quad . \quad (9)$$

IV.

Formula for Eccentric Loading.

When a column carries a load eccentrically, we find the maximum stress to be the sum of the stresses due to buckling and direct compression, as found in Ritter's formula, and the stress due to eccentric loading. We have for the first stress

$$f=\frac{P'}{A}\left(1+\frac{f}{m\pi^2E}\frac{l^2}{\rho^2}\right),$$

and for the second

$$f=\frac{P'}{A}\frac{ae}{\rho^2},$$

where a = eccentricity of load. The sum of these gives

$$f=\frac{P'}{A}\left(1+\frac{f}{m\pi^2E}\frac{l^2}{\rho^2}+\frac{ae}{\rho^2}\right),$$

from which the breaking load is found to be

$$P'=\frac{fA}{1+\frac{f}{m\pi^2E}\frac{l^2}{\rho^2}+\frac{ae}{\rho^2}}. \quad \ldots\ldots \quad (10)$$

In the above formula (10), the value of a is equal to the sum of the distance of the load from the column plus the deflection due to the load. This value of (a) for the several end conditions is as follows:

For Fig. 35$_1$ $=\frac{1.2Pl^2x}{9.6EI-Pl^2}$, approximately;

" Fig. 35$_2$ $=\frac{0.6Pl^2x}{9.6EI-Pl^2}$;

" Fig. 35$_3$ $=\frac{0.8Pl^2x}{9.6EI-Pl^2}$;

" Fig. 35$_4$ $=\frac{2.4Pl^2x}{9.6EI-Pl^2}$;

where x = distance of load from the column. The first of the above values for the deflection is found in Merriman's "Mechanics."

CHAPTER IX.

STRESSES IN THIN CYLINDERS AND IN RIVETED JOINTS.

I.

Stresses in Thin Cylinders for Internal and External Pressure.

Consider a thin cylinder of diameter $=D$, length $=L$, thickness of material $=t$, under an internal pressure of P lbs. per square inch.

The tensile stress per unit area along any longitudinal section will be equal to the resultant internal pressure acting normal to the section, divided by the area of the section.

$$\text{The stress} = \int_0^{\pi} pL\frac{D}{2}d\phi \sin\phi = pDL;$$

$$\text{The area} = 2tl;$$

whence

$$\text{Stress per unit area} = f_t = \frac{PD}{2t} = 0.5\frac{PD}{t}. \quad (1)$$

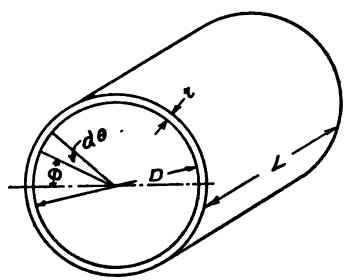

FIG. 38

The stress per unit area along any circumferential seam is found in the same manner.

$$\text{The stress} = \frac{\pi D^2}{4}p;$$

$$\text{The area} = \pi Dt;$$

whence

$$\text{Stress per unit area} = f_t' = \frac{PD}{4t} = 0.25\frac{PD}{t}. \quad . \quad . \quad (2)$$

It will be seen then that the intensity of stress along a longitudinal seam is twice the intensity of stress along a circumferential seam. Sometimes pipes are made spiral-riveted. Why this is done may be seen from the following discussion.

Let the spiral be at 45°, then the tensile stress per unit area along this seam is

$$f_t'' = \frac{\sqrt{\left(\frac{PD}{2t}\right)^2 t + \left(\frac{PD}{4t}\right)^2 t}}{\sqrt{2}\,t},$$

where $\sqrt{2}\,t$ = area of unit section along seam at 45° when longitudinal and transverse sections are taken as t. This reduces to

$$f_t'' = 0.395\frac{PD}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a perfectly round cylinder under external pressure the intensity of stress is the same in amount but opposite in sign. But because of the fact that cylinders are more or less imperfect, they are liable to collapse under external pressure. The design of such cylinders and tubes must be made, then, from empirical formulæ derived from tests.

Fairbairn performed a number of tests on tubes under external pressure, and from the data obtained the following formula:

$$p' = 9{,}672{,}000\frac{t^{2.19}}{ld}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which p' = uniform external pressure at which collapse occurred;
t = thickness of material;
l = length;
d = diameter.

Unwin recommends for tubes not less than $\frac{3}{8}''$ thick

$$p' = 9{,}672{,}000\frac{t^2}{ld}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In order to strengthen flues or cylinders, collapse-rings are used, and in some cases the flues are corrugated.

Lloyds and U. S. Board of Supervising Inspectors of Steam Vessels use

$$p=\frac{1{,}075{,}200t^2}{ld}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

whence

$$t=\frac{\sqrt{ldp}}{1037} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

when p = working pressure.

For collapse-rings of specified dimensions l is taken as the distance between such rings. For corrugated furnace-flues, where the corrugations are not less than $1\frac{1}{2}''$ deep and $8''$ between centers and the plain portions at the end not over $9''$, the U. S. B. S. I. use

$$p=\frac{14{,}000t}{d}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

II.

Stresses in Riveted Joints.

NOTATION.

t = thickness of plate;
d = diameter of rivet;
p = pitch of rivets;
n = number of rivets in single shear;
m = " " " " double shear;
f_t = fiber stress in tension;
f_s = " " " single shear;
f_s' = " " " double shear;
f_b = " " " crushing (single shear);
f_b' = " " " " (double shear);

E_s = efficiency in shear;
E_c = efficiency in crushing;
E_t = efficiency in tension;
E_m = efficiency of joint (maximum);
P = pressure in pounds per square inch;
D = diameter of vessel.

The following values of stress may be used:

Single-shear Rivets.	f_s.	f_b.
Iron plates and rivets.....	40,000	65,000
Steel " " "	50,000	90,000

Double-shear Rivets.	f_s'.	f_b'.
Iron plates and rivets.....	35,500	80,000
Steel " " "	45,000	110,000

f_t may be taken at 40,000 for iron and 55,000 for steel.

The factors entering into the design of riveted joints are briefly:

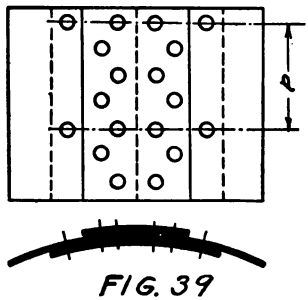

FIG. 39

(*a*) Strength; that is, the rivets must safely transmit the stress between the pieces they unite.

(*b*) Economy; since the rivets may fail in several ways, all liabilities for failure should be equal.

The ways in which riveted joints may fail are:

1st. Shearing of rivets.
2d. Crushing of rivet and plate.
3d. Tearing of plate between rivet-holes.

The strength of the joint in these different respects may be expressed as

$$S=\frac{n\pi d^2}{4}f_s+\frac{2m\pi d^2}{4}f_s'$$

$$=\frac{\pi d^2}{4}(nf_s+2mf_s'); \quad \ldots\ldots \quad (1)$$

$$C=ndtf_c+mdtf_c'$$

$$=dt(nf_c+mf_c'). \quad \ldots\ldots \quad (2)$$

$$T=(p-d)tf_t, \quad \ldots\ldots \quad (3)$$

and the resistance of the solid unit strip may be expressed

$$P=ptf_t.$$

The most economic values of d, t, and p will be those which will give equal values of S, C, and T.

These may be found as follows:

Let $S=C$, and we have

$$\frac{\pi d^2}{4}(nf_s+2mf_s')=dt(nf_c+mf_c'),$$

whence

$$dt=\frac{\frac{\pi d^2}{4}(nf_s+2mf_s')}{nf_c+mf_c'}. \quad \ldots\ldots \quad (4)$$

Let $T=S$, and we have

$$(p-d)tf_t=\frac{\pi d^2}{4}(nf_s+2mf_s'),$$

whence

$$ptf_t=\frac{\pi d^2}{4}(nf_s+2mf_s')+dtf_t=P. \quad \ldots\ldots \quad (5)$$

The efficiency of a joint is the ratio of the strength of the original strip to the riveted strip.

For maximum efficiency we must have $T=S=C$, which gives

$$E_m=S/P=T/P=C/P.$$

Substituting for S/P, we have

$$E_m=\frac{\frac{\pi d^2}{4}(nf_s+2mf_s')}{\frac{\pi d^2}{4}(nf_s+2mf_s')+dtf_t};$$

and substituting for dt from eq. (4) and eliminating, we have

$$E_m=\frac{1}{1+\frac{f_t}{nf_c+mf_c'}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From equation (4) we find the diameter of rivet for maximum efficiency

$$d=\frac{4(nf_c+mf_c')}{\pi(nf_s+2mf_s')}t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and from equation (5) we have the pitch for maximum efficiency

$$p=\frac{\pi d^2}{4}\left(\frac{nf_s+2mf_s'}{tf_t}\right)+d. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It can be seen from the above equations that the diameter of rivet and also the pitch for maximum efficiency can be found when the form of joint, giving the number of rivets in single and double shear per unit strip, and the thickness of the plate are known.

The form of the joint can easily be determined for any case

by comparison with those which usually exist in practice. The thickness of the plate for a boiler may be expressed as

$$t=\frac{PD}{2E_m f_t}.$$

Substituting for E_m, we have

$$t=\frac{PD(nf_c+mf_c'+f_t)}{2(nf_c+mf_c')f_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This gives the thickness of plate for maximum efficiency for any form of joint when the pressure P and diameter D are known and the proper values for allowable stresses are assumed. Having found the thickness of plate required, the diameter of rivet may be found from eq. (7), and with d and t thus found, P may be found from eq. (8).

To illustrate, let it be proposed to design the longitudinal seam of a boiler whose diameter is 60 inches, to carry a pressure of 200 lbs. per square inch gauge. The first step will be to decide on the form of the joint. From usual practice under these conditions, a joint illustrated in Fig. 39 is assumed, having 1 rivet in single shear and 4 rivets in double shear per unit strip. Letting this joint be designed with a factor of safety of 5 and assuming stresses as found on page 46, we have

$$t=\frac{200\times 60(60{,}000+288{,}000+50{,}000)}{2(60{,}000+288{,}000)50{,}000}\times 5$$
$$=0.68=\tfrac{11}{16}''$$

and

$$d=\frac{4(60{,}000+288{,}000)}{\pi(36{,}000+256{,}000)}\times 0.68$$
$$=1''$$

for

$$p=\frac{\pi(36{,}000+256{,}000)}{4(0.68\times 50{,}000)}+1$$
$$=7\tfrac{5}{8}''.$$

To analyze the efficiency of existing riveted joints, the following will be required:

$$E_t = \frac{(p-d)tf_t}{ptf_t} = \frac{p-d}{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$E_s = \frac{\pi d^2}{4}\frac{(nf_s + 2mf_s')}{ptf_t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$E_c = \frac{dt(nf_c + mf_c')}{ptf_t} \cdot = \frac{d}{p}\left(\frac{nf_c + mf_c'}{f_t}\right). \quad . \quad . \quad . \quad (12)$$

The lowest one of the above will of course be the actual efficiency of the joint.

CHAPTER X.

STRESSES IN BOLTS AND KEYS.

I.

Stresses in Bolts.

Stresses in bolts may be caused by any combination of the following things:

(*a*) Screwing up.

(*b*) External dead load.

(*c*) Shock.

To resist shock the bolt is reduced in body to equal the area at the base of the threads. Concerning the first two causes of stress, Professor Barr writes: "When the conditions are such that the nut is not apt to be screwed up hard, that is, when the initial load may be safely neglected, design for the external load alone.

The following suggestions may safely serve as a guide in practical problems involving the resultant load on bolts, when the initial load due to screwing up is apt to be considerable:

(*a*) If the bolt is manifestly very much more yielding than the connecting members, design the bolt simply for the initial load or for the external load, whichever is greater.

(*b*) If the probable yield of the bolt is from one-half to once that of the connecting members, consider the resultant load as the initial load plus from one-fourth to one-half the external load.

(*c*) If the yield of the connecting members is probably four or five times that of the bolts, take the resultant load as the initial load plus about three-fourths the external load.

(*d*) In case of extreme relative yielding of the connecting members, the resultant load may be assumed at nearly the sum of the initial and external loads."

Mr. Harvey D. Williams made an investigation on the strength of U. S. standard bolts, and reported the same in the Journal of the Amer. Soc. of Naval Engineers, Vol. XIII, No. 2. In his investigation he considers the stress due to load and also from the torque due to screwing up. Bolts of various materials are considered, among which is wrought iron with ultimate strength at 50,000 lbs. For this we have for a factor of safety of 7 and considering the efficiency of the screw the following values:

Diameter.	Working Load.	Diameter.	Working Load.
$\frac{1}{4}$	143	$1\frac{1}{8}$	3,990
$\frac{5}{16}$	247	$1\frac{1}{4}$	5,180
$\frac{3}{8}$	376	$1\frac{3}{8}$	6,110
$\frac{7}{16}$	519	$1\frac{1}{2}$	7,540
$\frac{1}{2}$	704	$1\frac{5}{8}$	8,820
$\frac{9}{16}$	912	$1\frac{3}{4}$	10,150
$\frac{5}{8}$	1140	$1\frac{7}{8}$	12,000
$\frac{3}{4}$	1725	2	13,400
$\frac{7}{8}$	2410	$2\frac{1}{4}$	17,800
1	3170	$2\frac{1}{2}$	21,900

The hollow bolt is more efficient than the solid bolt of the same area, so that the above values will be on the safe side for hollow bolts.

II.

STRESSES IN KEYS.

Machine keys are usually proportional empirically. The following rules are found to give good results:

Jones and Laughlin use

$$W = D/4;$$

$$T/2 = \tfrac{5}{16}W + \tfrac{1}{32}.$$

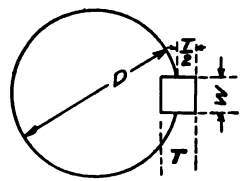

FIG. 40

National Tube Works use

$$W = D/4 + \tfrac{1}{8};$$

$$T = D/11 + 0.16.$$

The U. S. Navy uses these proportions:

D	*W*	*T*	*D*	*W*	*T*	*D*	*W*	*T*
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
$\frac{1}{2}$	$\frac{7}{32}$	$\frac{5}{16}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{16}$	$6\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{4}$
$\frac{5}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	$3\frac{1}{2}$	$\frac{13}{16}$	$\frac{7}{16}$	7	$1\frac{7}{16}$	$\frac{13}{16}$
$\frac{3}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$3\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	$7\frac{1}{4}$	$1\frac{1}{2}$	$\frac{13}{16}$
$\frac{7}{8}$	$\frac{9}{32}$	$\frac{7}{32}$	4	$\frac{7}{8}$	$\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{9}{16}$	$\frac{7}{8}$
1	$\frac{5}{16}$	$\frac{7}{32}$	$4\frac{1}{4}$	$\frac{15}{16}$	$\frac{9}{16}$	$7\frac{3}{4}$	$1\frac{9}{16}$	$\frac{7}{8}$
$1\frac{1}{8}$	$\frac{11}{32}$	$\frac{1}{4}$	$4\frac{1}{2}$	1	$\frac{9}{16}$	8	$1\frac{5}{8}$	$\frac{7}{8}$
$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$4\frac{3}{4}$	1	$\frac{9}{16}$	$8\frac{1}{4}$	$1\frac{5}{8}$	$\frac{15}{16}$
$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	5	$1\frac{1}{16}$	$\frac{5}{8}$	$8\frac{1}{2}$	$1\frac{3}{4}$	$\frac{15}{16}$
$1\frac{1}{2}$	$\frac{13}{32}$	$\frac{1}{4}$	$5\frac{1}{4}$	$1\frac{1}{8}$	$\frac{5}{8}$	$8\frac{3}{4}$	$1\frac{3}{4}$	$\frac{15}{16}$
$1\frac{3}{4}$	$\frac{7}{16}$	$\frac{5}{16}$	$5\frac{1}{2}$	$1\frac{3}{16}$	$\frac{5}{8}$	9	$1\frac{3}{4}$	1
2	$\frac{1}{2}$	$\frac{5}{16}$	$5\frac{3}{4}$	$1\frac{3}{16}$	$\frac{11}{16}$	$9\frac{1}{4}$	$1\frac{7}{8}$	1
$2\frac{1}{4}$	$\frac{9}{16}$	$\frac{5}{16}$	6	$1\frac{1}{4}$	$\frac{11}{16}$	$9\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{1}{16}$
$2\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{8}$	$6\frac{1}{4}$	$1\frac{5}{16}$	$\frac{3}{4}$	$9\frac{3}{4}$	2	$1\frac{1}{16}$
$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{8}$	$6\frac{1}{2}$	$1\frac{3}{8}$	$\frac{3}{4}$	10	2	$1\frac{1}{16}$
3	$\frac{11}{16}$	$\frac{7}{16}$						

The stresses to which a key is subjected are shearing and crushing. Thus, if l=length, w=width, and T=thickness, the

shearing strength will be $l \times w \times f_s$, and the crushing strength will be $l \times T/2 \times f_c$. The force producing the above stresses will be approximately $\frac{Pd}{R}$, where P = force at rim of pulley or gear, d = diameter of shaft, R = radius of pulley or gear. As stated before, however, keys are never analyzed for stress, but are proportioned empirically.

When the shaft is 8″ or more, it is advised to use two keys at 90° or 120° apart.

CHAPTER XI.

BEARINGS.

The several types of journal bearings may be conveniently grouped as follows:

1. The plain cylindrical, as used in main bearings for steam-engines, railway axles, etc.

2. Thrust-bearings may be divided into two classes: pivot- or step-bearings and collar-bearings.

3. Roller-bearings.

4. Ball-bearings.

5. Sliding-bearings.

I.

Plain Cylindrical Bearings.

PROPORTIONS.

The torsion and bending moments existing in the shaft and the rubbing of the journal over the bearing will indicate that the journal must be proportioned for *strength*, *stiffness*, and *wear*.

The *strength*, that is the *allowable fiber stress*, a journal shall have and the *allowable deflection* will be factors determining the *diameter*.

The wear of the journal, which is dependent on the *bearing pressure*, *speed*, and *lubrication*, will determine the length.

From experience previously gained from Chapters II and IV we are able to design the journal for strength and deflection. The length of the journal multiplied by the diameter will be known as the area of the journal.

For given ranges of speed the bearing pressure per square inch should be within given limits.

The only safe guide to these limits are the results of successful practice.

For the several cases arising in practice the following values are sometimes used:

(*a*) Continuous running-bearings, as railway-car axles, a value of 300 to 350 lbs. may be used.

(*b*) Crank-pins, where the pressure is reversed from one side to the other and the oil is less liable to be squeezed out, take for moderate-speed engines 500 lbs.; for slow-speed engines a higher value should be used, for high-speed engines a lower value.

The best data on engine-bearings is found in Professor Barr's paper on "Current Practice in Engine Proportions" before the A. S. M. E., May 1897.

He gives

Main journals:

$$d = C\sqrt[3]{\frac{\text{H.P.}}{N}}; \quad \ldots\ldots\ldots \quad (1)$$

$$l = Kd;$$

$$dl = M \times A \text{ (area of piston)};$$

where the constants have the values:

		Mean.	Max.	Min.
High Speed.				
(1) Center crank for each of two journals.	C	7.3	8.5	6.5
	K	2.2	3.0	2.0
	M	0.46	0.70	0.37
Low Speed.				
(2) Side-crank for one journal only.	C	6.8	8.0	6.0
	K	1.9	2.1	1.7
	M	0.56	0.64	0.46

rank-pin:

$$d=\frac{KA}{l}; \quad \ldots \ldots \ldots \quad (2)$$

$$l=C\frac{\text{H.P.}}{L}+B; \quad \ldots \ldots \ldots \quad (3)$$

L = length of stroke.

		Mean.	Maximum.	Minimum.
1	C	0.30	0.46	0.13
	K	0.24	0.44	0.17
	B	2.5″	2.5″	2.5″
2	C	0.6	0.8	0.4
	K	0.09	0.115	0.065
	B	2.0″	2.0″	2.0″

Cross-head pin:

$$d=\frac{l}{K}; \quad \ldots \ldots \ldots \quad (4)$$

$$l=\frac{CA}{d}; \quad \ldots \ldots \ldots \quad (5)$$

whence

$$d=\sqrt{\frac{CA}{K}}; \quad \ldots \ldots \ldots \quad (6)$$

$$l=\sqrt{CAK}. \quad \ldots \ldots \ldots \quad (7)$$

		Mean.	Maximum.	Minimum.
1	C	0.08	0.11	0.06
	K	1.25	2.0	1.0
2	C	0.07	0.10	0.054
	K	1.3	1.5	1.0

For marine engines the following values are sometimes used:

Crosshead-pins. ..	1000 to 1500	lbs. per square inch
Crank-pins.....	400 "	500 or 600 lbs. per square inch
Main bearings....	150 "	350 " 400 " " " " "
Guide-surfaces....	50 "	100 lbs. per square inch
Thrust-blocks....	40 "	80 or 100 lbs. per square inch

For stationary engines these values may for some cases be a little different. For guide-surfaces the pressure per square inch may be from 25 to 40 lbs. Crosshead-pins about the same. Crank-pins the same for center-crank engines, while for side-crank engines values between 900 to 1300 lbs. are sometimes used. Main bearings about the same for vertical engines, while for horizontal engines lower values are usually used.

For slow-moving machinery a large bearing pressure may be used. For instance, in punching-machines values as high as 2000 to 2500 lbs. per square inch are found.

The work absorbed in friction in plain cylindrical bearings may be found as follows:

Let P = total load on bearing in pounds;
d = diameter in inches;
l = length in inches;
N = revolutions per minute;
μ = coefficient of friction.

Then

$$\text{Work done per minute} = \frac{\mu P \pi d n}{12} \text{ foot-pounds,}$$

which reduces to

$$\frac{PdN\mu}{126{,}000} \text{ H.P.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

LUBRICATION.

The results of a number of experiments on lubricated surfaces indicate that the frictional resistance varies in the following manner:

- Speed
  - Rest and Reversal: Resistance much greater than for continuous motion.
  - Low Pressure: Directly.
  - High Pressure:
    - (*a*) Very great at low velocities.
    - (*b*) Becomes a minimum at 100 ft. per minute.
    - (*c*) Above 100 ft. per minute increases approximately as speed.
- Temperature: Depends more upon temperature than any other condition, decreasing as temperature increases to practical limits.
- Material of Journal: For bath lubrication there is little difference, but for meagre lubrication the material is quite important.
- The viscosity of the lubricant determines the maximum pressure at which the lubricant is squeezed out and seizing of the journal occurs.

The student is advised to consult the catalogues of engines and machine-tools and note the appliances for lubricating. It should be noted that the best place to introduce oil into a bearing is at points of least pressure in order that the oil may be easily carried to all parts of the bearing. If oil is introduced at the point of maximum pressure, it will not be able to get in and lubricate the bearing.

ADJUSTMENT.

No matter how good the lubricant may be, there can be no motion of parts over one another without wear. There must be some means, therefore, for taking up this wear before it becomes

a serious defect in the bearing. Play between journal and bearing reduces the accuracy of the machine, and may cause excessive strains from shocks in bearings where the pressure is reversed. It may also be well to note the fact that sometimes the proportions of journals are larger than strength would require in order to provide a larger wearing surface and thus insure accuracy.

In simple types of bearings a liner is sometimes placed between the cap and body of the bearing, and this is removed and replaced by a thinner one as wear occurs.

Other forms of bearings permit of adjustment by means of wedges operated by screws and secured by lock-nuts.

Greatest adjustment must of course be made in the direction of the greatest wear, so that when wear is decidedly in one or two directions the bearing must be designed for adjustment in these directions.

II.

THRUST-BEARINGS.

Thrust-bearings are usually horizontal or vertical. In the first case the pressure is taken up by a series of collars turned on the shaft. In the second case the pressure is taken up by the end of the shaft.

For thrust bearings Professor A. W. Smith recommends the following table as an approximate guide in the designing of thrust-bearings. The material used is wrought iron or steel and the bearing is of bronze or brass, babbitt metal being seldom used for this purpose. A bath lubricant is used. For cast-iron journals the values of the allowable pressures given should be divided by 2.

Mean Velocity of Rubbing Surface in Feet per Minute.	Allowable Unit Pressure in Pounds per Square Inch.
Up to 50	1000
50–100	600
100–150	350
150–200	100
Above 200	50

For different forms of thrust-bearings the student may consult "Machine Design," Part II, by Professor F. R. Jones.

III.

ROLLER-BEARINGS.

In order to reduce friction, bearings are made in which a number of rollers are placed between the journal and bearing. Very good results are obtained with this form of bearing for not too heavy service, and comparative tests show that the friction is very much less than with the plain form.

One difficulty experienced is the end thrust which results from the rollers inclining toward a helical path because they are not absolutely true. From tests it is found that the friction decreases with increase of both speed and load up to certain limits. One form of roller bearing is the Hyatt bearing. The rollers are made of ribbons of steel. The adjacent rollers are so placed that the spirals run in opposite directions, thus distributing the lubricant over the bearing surface. Very little lubricant is needed for roller-bearings, and it should be of liquid form.

Concerning the allowable pressure to be used on solid roller bearings we find the following in Jones' "Machine Design": "The safe load in pounds for a well-made solid roller journal-bearing with six or more rollers is

$$P = 100{,}000D^2\frac{NL}{3S} \text{ (for values of } S \text{ not less than 50 ft. per min.),} \quad (9)$$

in which D = diameter of roller, inches;
L = length of one roller, inches;
N = number of rollers;
P = total safe load on bearing, pounds;
S = linear velocity of convex bearing surface relatively to concave bearing surface, feet per minute.

For values of S less than 50 ft. per minute the pressure may be taken the same as for $S=50$. For exceedingly slow speeds the pressure may be taken somewhat higher, however. This equation is based upon the assumption that the safe load per inch of total effective length of rollers is $2000D^2$ lbs., considering one-third of the rollers under pressure."

IV.

BALL-BEARINGS.

In a great many cases where light pressures are to be used it is found that ball-bearings can be used with advantage. The load that can be used on a ball-bearing is a factor of the number and size of the balls and the speed. The crushing strength of the ball cannot be considered in determining the safe load to be employed. Experiments by Professor Benjamin have shown, for instance, that a $\frac{1}{4}$-inch ball has a crushing strength of 15,000 lbs., whereas it can only safely sustain in service about 100 lbs. and give good results. In designing ball-bearings the data obtained from successful practice is the best guide to use.

In regard to friction of ball-bearings Professor Goodman has found from experiments that it varies:

1. Directly as the load.
2. Is independent of the speed.
3. Is independent of the temperature.
4. The friction of rest is but very slightly greater than the friction of motion.
5. Is not reduced by lubrication in a clean well-designed bearing.

Ball-bearings are made so as to have 2-, 3-, or 4-point contact.

For 2-point contact the race is usually a circular groove of a little larger radius than the radius of the ball.

For 3-point contact the race is constructed as shown in Fig. 41. The sides of the groove (*ab*) and (*ac*) are tangent to the ball at the points where the ball is intersected by the line *AB*. This places the points of contact on the surface of a cone whose apex is at *A*, thus giving the ball a pure rolling motion about the axis *A*, provided the alignment is perfect. Four-point contact is constructed after the same manner.

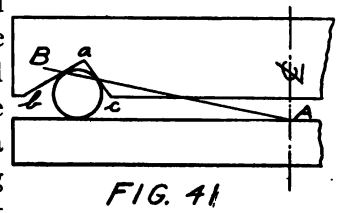

FIG. 41

It will be found, I think, that 2-point contact as a rule will give better service, since any very small error in alignment will not much affect the bearing. For the 3- and 4-point contacts it can be seen that there must be perfect alignment to insure pure rolling motion.

V.

SLIDING-BEARINGS.

Sliding-bearings are used on planers, shapers, etc., where a rectilinear or other than rotary motion is desired. The methods of adjustment for wear and methods of lubrication are several, and will be considered in the lectures. The student is advised to examine the sliding-bearings on the planers and shapers in the machine-shop, and also to consult the machine tool catalogues.

CHAPTER XII.

STRESSES IN BELTS.

The stresses occurring in a belt transmitting power may be analyzed as follows:

Let Fig. 42 represent a belt passing around a pulley, and Fig. 43 any portion of such a belt.

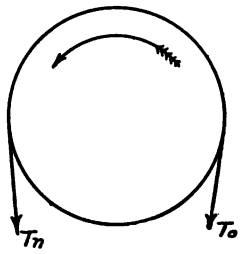

FIG 42

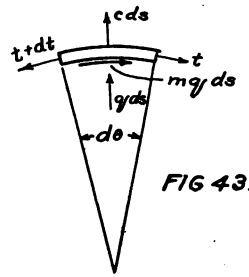

FIG 43.

It will appear that, in order that the piece shown in Fig. 43 be in equilibrium, the following relations must hold:

$$qds+cds=t\sin\frac{d\theta}{2}+(t+dt)\sin\frac{d\theta}{2},$$

which, neglecting small products and letting $\sin\frac{d\theta}{2}=\frac{d\theta}{2}$, reduces to

$$qds=td\theta-cds,$$

where t=unit tension at any point in the belt;

c = centrifugal force per unit length

$$= \frac{12wv^2}{gr} \times a$$

$$= \frac{12wv^2}{gr} \times \frac{t}{t_w},$$

where t_w = working tension per unit area;

a = unit area;

v = velocity of belt in feet per second;

r = radius of pulley in feet;

w = weight per unit volume.

Substituting this value of c and also $ds = rd\theta$, we have

$$qds = td\theta - 12\frac{wv^2}{gt_w}td\theta$$

$$= td\theta(1 - x),$$

where $x = \frac{12wv^2}{gt_w}$.

It also appears that

$$dt = \mu qds,$$

$$qds = \frac{dt}{\mu},$$

where μ = coefficient of friction.

Substituting this in the above equation, we find

$$\frac{dt}{t} = \mu(1 - x)d\theta,$$

$$\int_{t_0}^{t_n} \frac{dt}{t} = \int_0^{\theta} \mu(1 - x)d\theta,$$

or

$$\log_e \frac{t_n}{t_0} = \mu(1 - x)\theta,$$

$$\text{common log } \frac{t_n}{t_0} = 0.4343(1 - x)\theta, \quad \ldots\ldots \quad (1)$$

where θ = arc of contact in radians = 0.0175α;
α = arc of contact in degrees.

For slow speeds—under 2000 feet per minute—x is small enough to be neglected and we have

$$\log_e \frac{t_n}{t_0} = \mu\theta,$$

or

$$\text{common log } \frac{t_n}{t_0} = 0.4343\mu\theta = 0.0076\mu\alpha, \quad . \quad . \quad . \quad (2)$$

where t_n = unit tension on tight side;
t_0 = unit tension on slack side.

The turning force per square inch of belt is

$$p = t_n - t_0.$$

The total turning force required to transmit any given horse-power in terms of the velocity of the belt in feet per minute is

$$P = \frac{33{,}000 \times \text{H.P.}}{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The area of the belt is

$$A = \frac{P}{p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The total tensions are

$$T_n = At_n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$T_0 = T_n - P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The working strength of leather belting may be taken at 70 lbs. per inch of width for single belts.

The weight of leather belting is about 0.035 lb. per cubic inch.

For these values of strength and weight the curve in Fig. 44 is drawn.

The coefficient of friction of leather belting on ordinary wooden or iron pulleys is about 0.3 to 0.4.

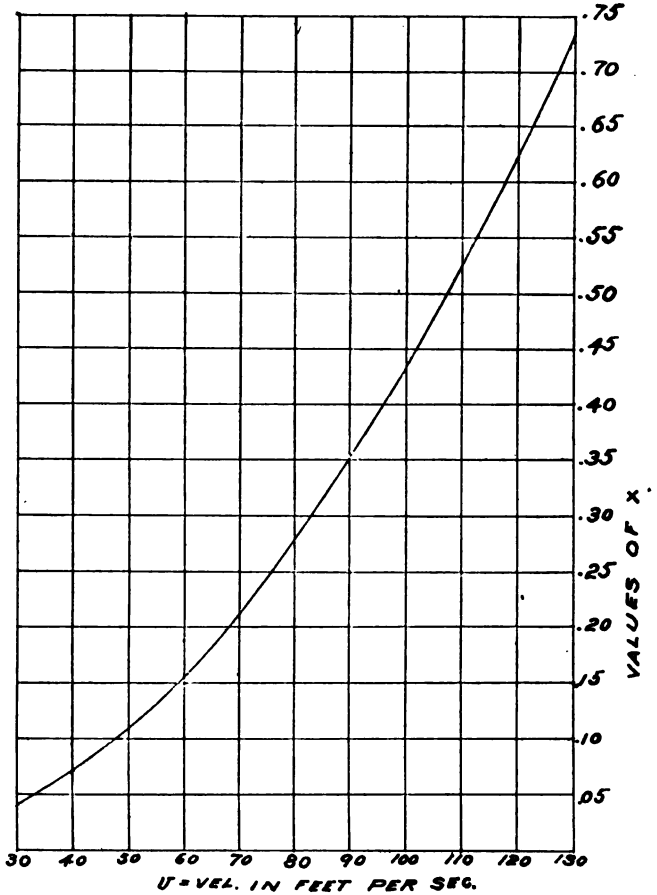

FIG. 44.

The performance of belts in practice has been carefully studied by Mr. F. W. Taylor and reported to the A S. M. E., which report forms part of Vol. XV of the transactions of that society. Among other things Mr. Taylor recommends:

1. The belt speed for maximum economy should be from 4000 to 4500 ft. per minute.

2. Belts are more durable and work more satisfactorily made narrow and thick than wide and thin.

3. It is safe and advisable to use a double belt on a pulley 12 inches in diameter or larger; a triple belt on a pulley 20 inches in diameter or larger; a quadruple belt on a pulley 30 inches in diameter or larger.

4. The ends of the belt should be fastened together by splicing and cementing, instead of lacing or using hooks or clamps of any kind.

5. Belts should be cleaned and greased every five or six months.

For rope driving the same formulas may be applied and the coefficient of friction taken from 0.3 to 0.5 for grooves from 45° to 30°, and the working strength of the ropes is assumed at $200d^2$, where d = diameter of rope.

For ordinary practice the size of rope for power transmission is from 1″ to 1¾″. Larger ropes are used for large powers.

When ropes are used instead of belts there are two systems usually used. One, in which the rope throughout the entire drive is continuous—one rope—passing around the several pulleys in the drive, with the last stretch passing over a tension-pulley having a weight attached and arranged to keep the tension in the rope as nearly constant as practicable. This is known as the American system.

The other system, known as the English system, is made up of individual ropes, each running in its own groove in the pulleys.

CHAPTER XIII.

THEORY OF ELASTICITY. STRESSES IN THICK CYLINDERS UNDER INTERNAL PRESSURE.

I.

Theory of Elasticity.

The following simple explanation is given to aid the student in the solution of the problems which follow.

It will be noted that when a test-piece is subject to tension it not only elongates but also contracts laterally, and when subject to compression the reverse is true, that is, it grows shorter longitudinally and bulges out laterally.

From a number of experiments it has been found that the amount of lateral contraction or expansion bears a definite ratio to the elongation or shortening, as the case may be, for the same materials. The ratio of this lateral deformation per unit change of length, which may be denoted as ε, has been found for wrought iron and steel to have a value of about $\frac{1}{4}$ or $\frac{1}{3}$, and for cast iron as much as $\frac{1}{4}$ or less.

One of the earliest investigators of this phenomenon was Poisson, and this ratio is sometimes known as *Poisson's Ratio.*

To illustrate the above, consider a bar of steel under tension due to a force T. If the original length is l, and the total elongation is λ, then $\frac{\lambda}{l} = S =$ the elongation per unit length, whence the lateral contraction resulting is εS. For practical purposes Hooke's Law is accepted, namely, that within the elastic limit

stress and strain are proportional, so that for each unit of elongation there is required a definite tension.

Inasmuch, then, as we find a definite unit elongation per unit stress, it is reasonable to assume that the *lateral deformation is indicative of lateral strain*, and that its ratio to the longitudinal strain is some constant ϵ. When the force elongating the bar $=1000$ lbs., each side of the bar may be considered in a state of strain such as would be produced by a stress of $\frac{1}{4}\times 1000 = 250$ lbs. when ϵ is taken as $\frac{1}{4}$.

In accord with the above, if we have a bar as shown in the figure and subjected to stresses as indicated, it will be found that the strains in the bar along the three axes are equivalent to

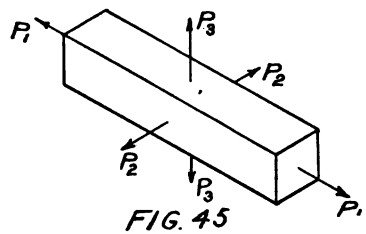

FIG. 45

$$T_1 = P_1 - \epsilon P_2 - \epsilon P_3;$$
$$T_2 = P_2 - \epsilon P_1 - \epsilon P_3;$$
$$T_3 = P_3 - \epsilon P_1 - \epsilon P_2.$$

If any of the stresses in the above were compressive, the proper signs of course would have to be used.

II.

Stresses in Thick Cylinders under Internal Pressure.*

Let Fig. 46 represent the cross-section of a thick cylinder under an internal pressure p_1; the internal radius is r_1, and the external radius is r_2. For analysis consider an elementary portion of the cylinder at a distance r from the center and let it have *unit length*. As shown in Fig. 46, the dimensions of the portion will be $rd\theta$, $(r+dr)d\theta$, and dr thick, and unit length as stated before. The forces holding this portion in equilibrium are as shown in the figure. Considering the forces qdr, we find they may be resolved as shown in Fig. 47. Thus we

* Mr. T. J. Rodhouse of the University of Missouri kindly assisted in the following solution for thick cylinders.

have the components (a) and (a) equal and opposite, hence balancing one another, and the forces (b) and (b) acting radial,

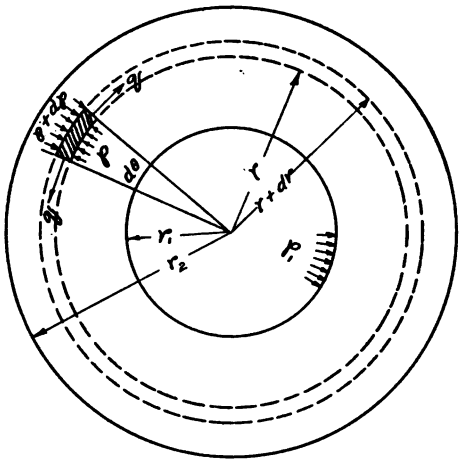

FIG. 46

which forces are equal to $qdrd\theta$. We have then the following equation for equilibrium:

$$(p+dp)\times(r+dr)d\theta - prd\theta = qdrd\theta,$$

whence is obtained

$$pdr + rdp = qdr$$

when the product $dpdr$ is dropped, since it is too small to be considered.

FIG. 47

The above equation may be written

$$d(pr) = qdr,$$

whence

$$\frac{d(pr)}{dr}=q, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which gives a relation between p and q.

Another relation between p and q is needed to effect a solution, and this relation may be found as follows:

Considering an elementary portion as shown in Fig. 48 and resolving the forces as shown, we have that the longitudinal strain is equal $-\varepsilon q+\varepsilon p$ or $-\varepsilon(q+p)$, and since this is taken at any cross-section, the longitudinal strain is a constant; hence

FIG. 48

$$q+p=(\text{a constant}), \text{ say } 2C_1, \quad . \quad . \quad (2)$$

where the 2 is used for convenience in integration, as shown later. We now have two relations between q and p as desired.

Substituting equation (2) in equation (1), we have

$$\frac{d(pr)}{dr}+p=2C_1,$$

whence

$$r\frac{dp}{dr}+p\frac{dr}{dr}+p=2C_1,$$

or

$$r\frac{dp}{dr}+2p=2C_1.$$

Multiplying both sides of the equation by r, we have

$$r^2\frac{dp}{dr}+2pr=2C_1r,$$

or

$$r^2dp+2prdr=2C_1rdr;$$

whence

$$d(pr^2)=2C_1rdr;$$

whence, upon integrating, we have

$$pr^2 = C_1 r^2 + C_2,$$

or

$$p = C_1 + \frac{C_2}{r^2}.$$

Substituting this in equation (2), we have

$$q = 2C_1 - \left(C_1 + \frac{C_2}{r^2}\right)$$

$$= C_1 - \frac{C_2}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

It now remains to find values for C_1 and C_2, which may be done as follows:

Let the internal pressure be p_1 and the external pressure be o. Whereupon at inside of cylinder

$$\left.\begin{matrix} p = p_1 \\ r = r_1 \end{matrix}\right\} \text{ and } \therefore\ p_1 = C_1 + \frac{C_2}{r_1^2}; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

at outside of cylinder

$$\left.\begin{matrix} p = o \\ r = r_2 \end{matrix}\right\} \text{ and } \therefore\ o = C_1 + \frac{C_2}{r_2^2}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substituting C_2 from (4) in (3), we have

$$p_1 = C_1 - \frac{C_1 r_2^2}{r_1^2},$$

whence

$$C_1 = \frac{p_1 r_1^2}{r_1^2 - r_2^2} = -\frac{p_1 r_1^2}{r_2^2 - r_1^2}.$$

Substituting C_1 from (4) in (3), we have

$$p_1 = -\frac{C_2}{r_2^2} + \frac{C_2}{r_1^2};$$

whence

$$-p_1 = C_2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

$$= C_2\left(\frac{r_1^2 - r_2^2}{r_1^2 r_2^2}\right),$$

from which

$$C_2 = -p_1\left(\frac{r_1^2 r_2^2}{r_1^2 - r_2^2}\right)$$

$$= p_1\left(\frac{r_1^2 r_2^2}{r_2^2 - r_1^2}\right).$$

Substituting these values of C_1 and C_2 in equation (2*a*) for q, we have

$$q = -\frac{p_1 r_1^2}{r_2^2 - r_1^2} - \frac{p_1 r_1^2 r_2^2}{(r_2^2 - r_1^2) r^2}$$

$$= -\frac{p_1 r_1^2}{r_2^2 - r_1^2}\left(1 + \frac{r_2^2}{r^2}\right).$$

The value of q on the inner surface where $r = r_1$ is

$$q_1 = -p_1\left(\frac{r_1^2 + r_2^2}{r_2^2 - r_1^2}\right), \quad \ldots\ldots \quad (5)$$

and the value of q on the outer surface where $r = r_2$ is

$$q_2 = -p_1\left(\frac{2r_1^2}{r_2^2 - r_1^2}\right). \quad \ldots\ldots \quad (6)$$

The stress on the inner surface is evidently the greater. Equation (5) is evidently the one to be used in designing thick cylinders. It may be transformed to read

$$r_2 = r_1\sqrt{\frac{q_1 + p_1}{q_1 - p_1}}, \quad \ldots\ldots \quad (7)$$

from which the radius of cylinder can be found for any size bore and given pressure and fiber stress.

The above theory is known as Lamé's theory and, of the several theories, is found to give results which are nearer actual conditions than any other.

Professor Goodman, in order to test this theory with the actual case, performed experiments on a thick cylinder, and by means of a delicate extensometer, placed at different depths in radial drilled holes, was able to read the stresses at these depths. The accompanying figure (49), arranged from his "Mechanics Applied to Engineering," gives the result of these tests.

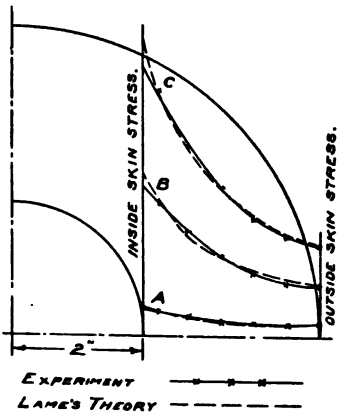

FIG. 49

A. For 0.33 tons per square inch internal pressure.
B. " 2.60 " " " " " "
C. " 4.74 " " " " " "

CHAPTER XIV.

STRESSES IN HOOKS.

The theory of hooks as usually given in text-books has not proven satisfactory. Very important factors materially affecting

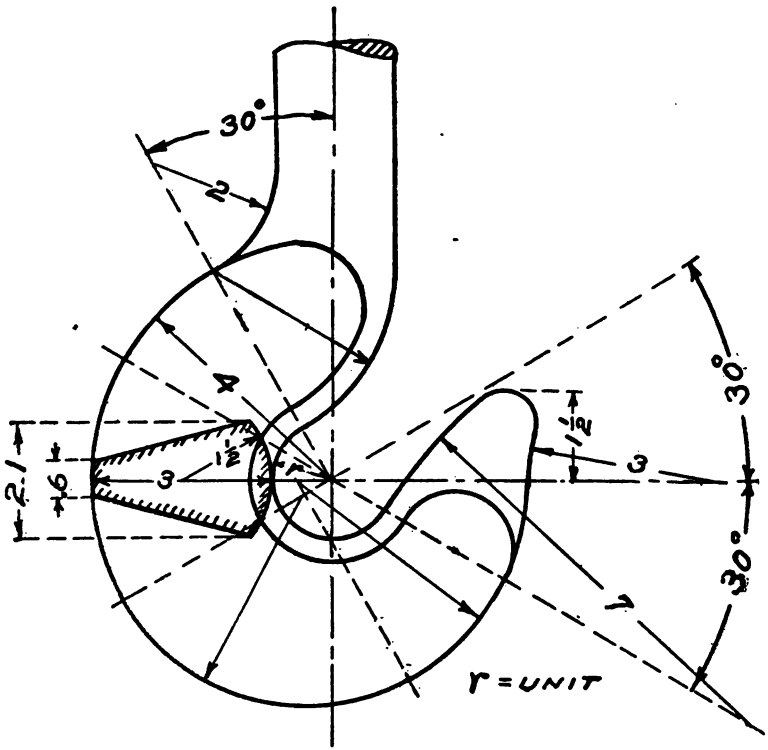

FIG. 50.

the stresses have been left out. The most complete theory and one justified by experiment has recently been developed by Mr.

E. S. Andrews and Prof. Karl Pearson of the University College, London. This theory and extracts from the experiments conducted to prove it are given in the following pages. This is essentially as found in the original paper with the exception of Figs. 52 and 53, which are added for clearness, and the application to the design of hooks of the form shown in Fig. 50 and in general.

In Fig. 51, *ABC* represents the horizontal section to which

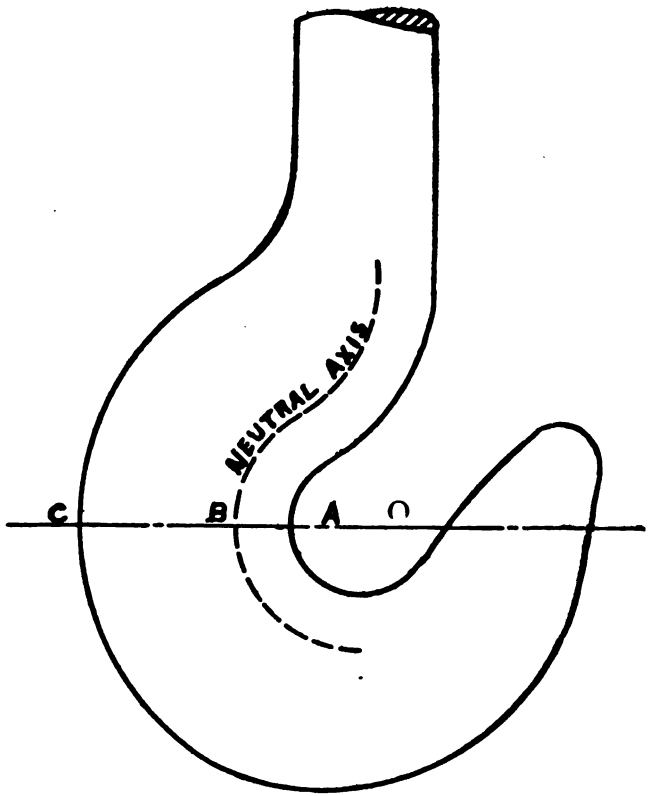

FIG. 51.

the investigation is confined, since it is the most dangerous section.

It is assumed that the center of curvature for the extrados at

C and the intrados at A is sensibly at o, which is a very close approximation for hooks as they exist. Also that the curvature changes very slightly in the neighborhood of ABC. It is further assumed that there is no shear on the section ABC, whence in the immediate neighborhood there would be no distortion of the section, whereupon the section after strain may be considered to remain plane. Symbols and letters with the subscript o attached to them will refer to quantities and points on the unstrained hook. With no subscript they will refer to the strained hook. As shown in Fig. 54, which is a cross-section of the hook at the horizontal section ABC, YY is the gravity axis and y represents the distance of any elementary area (α) from the axis.

O is the center of curvature such that $OB=\rho$. A = area of the entire section.

In Fig. 52 dashed letters will refer to corresponding points on a section made by passing a plane through a line through O parallel to YY, and this plane being in the immediate neighborhood of the section ABC. Let S_y represent the unit stretch of fibers situated at a distance y from YY, whereupon the tension in such a fiber will be

$$T_y=ES_y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

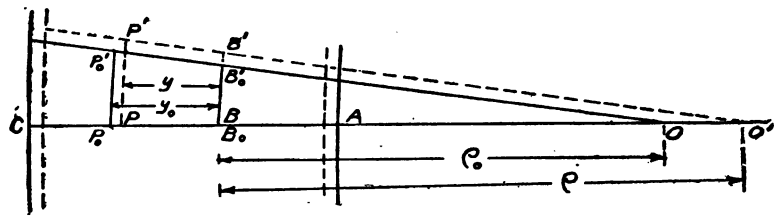

FIG. 52.

Referring to Fig. 52 we see that

$$S_y=\frac{PP'-P_0P_0'}{P_0P_0'},$$

$$PP'=P_0P_0'(1+S_y).$$

Also,

$$\frac{PP'}{BB'}=\frac{\rho+y}{\rho},$$

and

$$\frac{P_0P_0'}{B_0B_0'}=\frac{\rho_0+y_0}{\rho_0}.$$

Now if S_b is the stretch at B, we have

$$BB'=B_0B_0'(1+S_b).$$

By combining the above equations we find

$$\left(1+\frac{y}{\rho}\right)(1+S_b)=\left(1+\frac{y_0}{\rho_0}\right)(1+S_y),$$

from which

$$\frac{1+\frac{y}{\rho}}{1+\frac{y_0}{\rho_0}}=\frac{1+S_y}{1+S_b}=1-S_b+S_y,$$

or

$$S_y=S_b+\frac{\frac{y}{\rho}-\frac{y_0}{\rho_0}}{1+\frac{y_0}{\rho_0}}, \quad \ldots\ldots\ldots \quad (2)$$

whence

$$(S_y-S_b)\left(1+\frac{y_0}{\rho_0}\right)=\frac{y}{\rho}-\frac{y_0}{\rho_0}.$$

Differentiating the above, we have

$$\delta S_y\left(1+\frac{y_0}{\rho_0}\right)+(S_y-S_b)\frac{\delta y_0}{\rho_0}=\frac{\delta y}{\rho}-\frac{\delta y_0}{\rho_0}. \quad \ldots \quad (3)$$

The terms δy and δy_0 represent small transverse shortening of the fibers as shown in Fig. 53, and a relation between this transverse shortening and the corresponding transverse stress

may be obtained as follows: Let a fiber of thickness δy_0 before stress, at a distance y_0 from the gravity axis, have after stress a

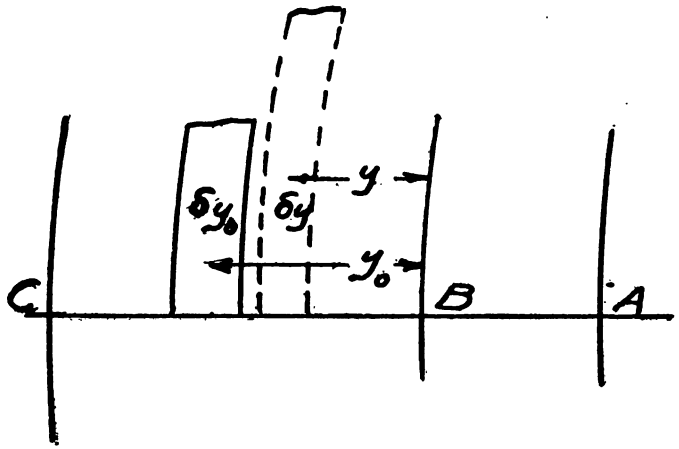

FIG. 53.

thickness δy and be at a distance y from the gravity axis. Whereupon we find the transverse shortening equal to

$$\frac{\delta y_0 - \delta y}{\delta y_0}.$$

But from the theory of elasticity we know this corresponds to a ~~stress~~ equal to ηS_y, where η = Poisson's ratio. Whereupon we have

$$\frac{\delta y_0 - \delta y}{\delta y_0} = \eta S_y,$$

or

$$\delta y = \delta y_0 (1 - \eta S_y).$$

If now we substitute this value of δy in the above differential equation, we have

$$\frac{\delta S_y}{\frac{1}{\rho} - \frac{1}{\rho_0} + \frac{S_b}{\rho_0} - S_y\left(\frac{1}{\rho_0} + \frac{\eta}{\rho}\right)} = \frac{\delta y_0}{1 + \frac{y_0}{\rho_0}}. \quad \ldots \quad (5)$$

If now we let $S_y\left(\frac{1}{\rho_0}+\frac{\eta}{\rho}\right)=S_y\left(\frac{1+\eta}{\rho_0}\right)$, which is practically true, we have upon integration

$$-\frac{\rho_0}{1+\eta}\log\left[\frac{1}{\rho}-\frac{1}{\rho_0}+\frac{S_b}{\rho_0}-S_y\left(\frac{1+\eta}{\rho_0}\right)\right]=\rho_0\log\left(1+\frac{y_0}{\rho_0}\right)+\log C,$$

which reduces to

$$\frac{1}{\rho}-\frac{1}{\rho_0}+\frac{S_b}{\rho_0}-S_y\left(\frac{1+\eta}{\rho_0}\right)=\frac{C}{\left(1+\frac{y_0}{\rho_0}\right)^{1-\eta}}. \quad . \quad . \quad . \quad (6)$$

Now when $S_y=S_b$, $y_0=0$. Whereupon we find the value for C:

$$C=\frac{1}{\rho}-\frac{1}{\rho_0}-\frac{\eta S_b}{\rho_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting the value of C in the equation above and solving for S_y, we find

$$S_y=\frac{S_b}{1+\eta}+\frac{\rho_0}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)-\frac{\frac{\rho_0}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_0}-\frac{\eta S_b}{\rho_0}\right)}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}},$$

or

$$S_y=S_b+\left\{\frac{\rho_0}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)-\frac{\eta S_b}{1+\eta}\right\}\left(1-\frac{1}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}}\right),$$

which gives as the tensile stress of any fiber at a distance y from the gravity axis

$$T_y=ES_b+E\left\{\frac{\rho_0}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)-\frac{\eta S_b}{1+\eta}\right\}\left(1-\frac{1}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}}\right). \quad (8)$$

If $S\alpha = A$, W = load, l = distance from load line to gravity axis, we have

$$W = S(T_y\alpha),$$
$$Wl = -S(T_y\alpha \times y_0).$$

If now we let

$$\beta = \frac{\rho_0}{+\eta}\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) - \frac{\eta S_b}{1+\eta},$$

$$A\gamma_1 = S\frac{\alpha}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}} = S\left[\left(\frac{\rho_0}{\rho_0+y_0}\right)^{1+\eta}\alpha\right], \quad . \quad . \quad . \quad . \quad (9)$$

$$A\gamma_2 = -S\frac{\alpha\frac{y_0}{\rho_0}}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}} = -S\left[\left(\frac{\rho_0}{\rho_0+y_0}\right)^{\eta}\alpha\right] + A\gamma_1, \quad . \quad (10)$$

we may write

$$W = EAS_b + E\beta A(1-\gamma_1),$$
$$Wl = -E\beta A\rho_0\gamma_2,$$

which gives

$$S_b = \frac{W}{EA} + \frac{Wl}{EA\rho_0}\frac{1-\gamma_1}{\gamma_2},$$

$$\beta = -\frac{Wl}{EA\rho_0\gamma_2}.$$

Equating the two values of β, we have

$$\frac{1}{\rho} - \frac{1}{\rho_0} = \frac{W}{EA}\left\{-\frac{l(1+\eta\gamma_1)}{\gamma_2\rho_0^2} + \frac{\eta}{\rho_0}\right\},$$

$$S_b = \frac{W}{EA}\left\{1 + \frac{l}{\rho_0}\frac{1-\gamma_1}{\gamma_2}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Which, when substituted in the equation for T_y, gives

$$T_y=\frac{W}{A}\left\{1+\frac{l}{\rho_0\gamma_2}\left(\frac{1}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}}-\gamma_1\right)\right\}. \quad . \quad . \quad . \quad (12)$$

When $y_0=h_1$, then $T_y=-C$, the maximum compressive stress, and when $y_0=h_2$, then $T_y=T$, the maximum tensile stress. Whereupon we have

$$C=\frac{W}{A}\left\{\frac{l}{\rho_0\gamma_2}\left(\gamma_1-\frac{1}{\left(1+\frac{h_1}{\rho_0}\right)^{1+\eta}}\right)-1\right\}, \quad . \quad . \quad . \quad (13)$$

$$T=\frac{W}{A}\left\{\frac{l}{\rho_0\gamma_2}\left(\frac{1}{\left(1-\frac{h_2}{\rho_0}\right)^{1+\eta}}-\gamma_1\right)+1\right\}. \quad . \quad . \quad . \quad (14)$$

As far as design is concerned, T, being the greater of the two, will interest us more.

This formula may be put in the form

$$\frac{W}{T}=\frac{A}{\frac{l}{\rho_0\gamma_2}\left[\left(\frac{\rho_0}{\rho_0-h_2}\right)^{1+\eta}-\gamma_1\right]+1}. \quad . \quad . \quad . \quad (15)$$

For any given cross-section γ_2 and γ_1 may be found as well as the other constants, whereupon the ratio of load to fiber stress may be computed. The solution of this equation may be made as follows: Let Fig. 50 represent the outline and proportions for a form of hook we wish to use. From this figure the factors l, ρ_0, A, and h_2 may be directly found when once we assign dimensions to the hook. For the solution of γ_1 and γ_2, in which η is taken as $\frac{1}{4}$, Mr. Andrews and Professor Pearson have devised the following graphic solution, Fig. 54.

Describe any parabola OV with O as a vertex and $OABC$ as an axis. Then describe any other parabola with O as a vertex

and an axis perpendicular to *OABC* as shown, letting *TW* be some convenient known length.

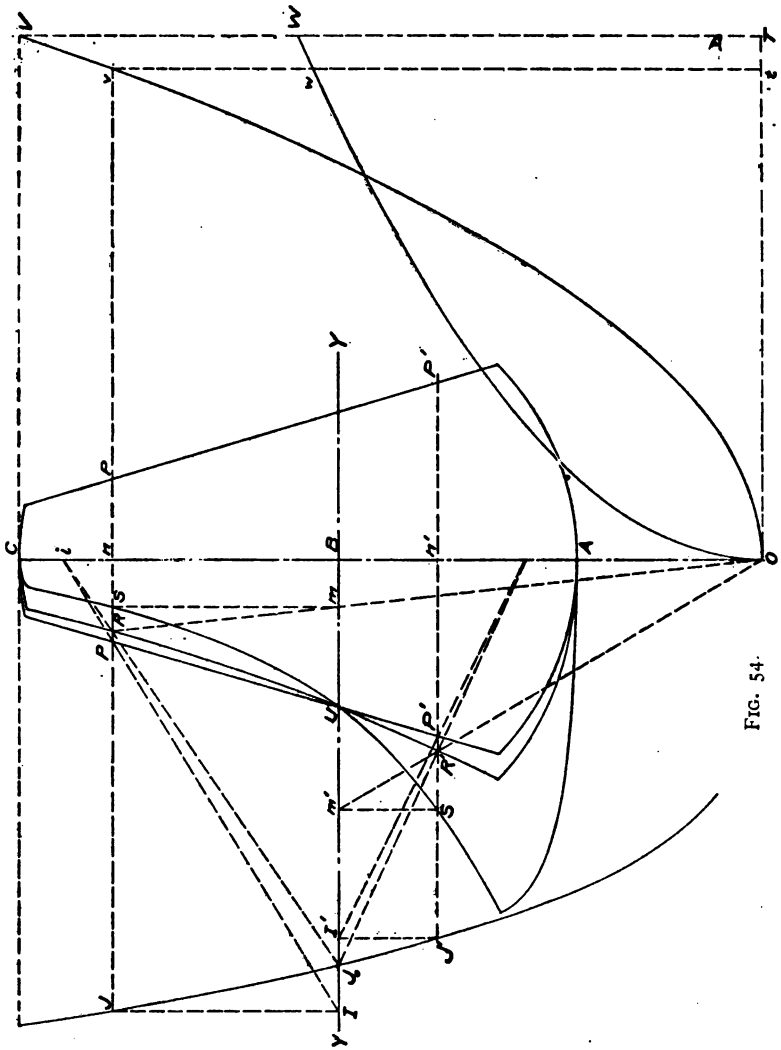

FIG. 54.

Let $CV = a$, $VT = d$, $TW = b$.

Let any line of the hook section PnP', parallel to YY, meet

the first parabola in v, and let vwt be parallel to $OABC$; then if $tw=Z$,

$$\frac{Z^2}{b^2}=\frac{ot}{a}, \quad \text{or} \quad \frac{Z^4}{b^4}=\frac{ot^2}{a^2}=\frac{nv^2}{a^2};$$

but

$$\frac{nv^2}{a^2}=\frac{on}{oC}=\frac{on}{d},$$

therefore

$$\frac{Z^4}{b^4}=\frac{on}{d},$$

or

$$Z=b\left(\frac{on}{d}\right)^{\frac{1}{4}}.$$

Let this curve be plotted as shown at OJ. ($Jn=wt$, etc.)

Let YY meet this curve in J_0 and PnP in J. Let the vertical JI through J meet yy in I. Let IP meet AC in i, and J_0i meet Pn in R; then

$$\frac{Rn}{Pn}=\frac{J_0B}{IB}=\frac{J_0B}{Jn}=\frac{oB^{\frac{1}{4}}}{on^{\frac{1}{4}}};$$

thus

$$Rn=Pn\left(\frac{\rho_0}{\rho_0+y_0}\right)^{\frac{1}{4}},$$

which gives the hook rigidity curve $CRUR_1A$.

Join RO and let it meet yB in m; draw ms perpendicular to PP; then

$$\frac{Sn}{Rn}=\frac{mB}{Rn}=\frac{\rho_0}{\rho_0+y_0},$$

$$Sn=\frac{\rho_0}{\rho_0+y_0}Rn=Pn\left(\frac{\rho_0}{\rho_0+y_0}\right)^{\frac{5}{4}},$$

which gives the second hook rigidity curve $CSUS_1A$.

We then find

$$\gamma_1 = \frac{\text{area } CSUS_1A}{\frac{1}{2}\text{ area of section}},$$

$$\gamma_2 = \frac{\text{area } CSUS_1A - \text{area } CRUR_1A}{\frac{1}{2}\text{ area of section}}.$$

For any *form of section* it will be evident that γ_1 and γ_2 are constants. Also upon inspection of the denominator of the right-hand member of equation (15) we find a constant for any *form of section.* Let us assume, then, a hook having proportions as given in Fig. 50. For any value of r, that is for any size of hook of this form, the factor mentioned above will be a constant and equation (15) will reduce to

$$\frac{W}{T} = \frac{A}{10.58}.$$

But A is a function of r found as follows:

The area of the trapezoidal section is computed to be $3.456r^2$ and the area of the circular segment is found to be $0.639r^2$. Whereupon $A = 4.095r^2$, approximately $4.1r^2$. This may be checked by finding the area with a planimeter and dividing by the radius squared. Fig. 54 was drawn for $r = 2$ and the area measured 16.38 square inches, whence $\frac{16.38}{4} = 4.095$ as a check.

We may now write

$$\frac{W}{T} = \frac{4.1r^2}{10.58},$$

$$r^2 = \frac{2.56W}{T},$$

$$r = \sqrt{\frac{2.56W}{T}} \quad \ldots\ldots\ldots \quad (16)$$

In this manner, then, equations similar to (16) may be derived for different forms of sections. According to the old theory as given in Unwin, etc., the tensile stress at the intrados is given as

$$T=\frac{W}{A}+\frac{WIl}{h_2},$$

where I = moment of inertia of the section.

It gives the stresses as the sum of the stress due to the direct pull and the stress due to cross-bending and does not take into account the radius of curvature which influences the stretch of the fiber, and hence the stress. That the new theory is far better than the old theory is seen from a number of experiments made by Mr. Andrews and Professor Pearson.

Of these, one was for a railroad-car coupling-hook and by actual test the material had its elastic limit at 21,000 lbs. per square inch, while the old theory gave a stress of 12,000 lbs. per square inch, and the new theory a stress of 23,000 lbs. per square inch, for W = load at elastic limit. The old theory errs a great deal in this case, while the new theory is on the safe side by a small margin. Other experiments made do not show such a marked difference except in one case where a T section was used, the old theory gave a stress of 18,700 lbs. per square inch at the elastic limit, the new theory a stress of 29,300 lbs. per square inch at the elastic limit, while the actual stress at the elastic limit was found by experiment to be 30,000 lbs. per square inch. It must be remembered that the theory applies only up to the elastic limit, since it is based on Young's Modulus E.

CHAPTER XV.

STRESSES IN REVOLVING RINGS AND DISKS.*

I.

STRESSES IN REVOLVING RINGS.

With the advent of the steam-turbine in modern engineering practice, this problem is quite an important one to be considered. The high speeds at which turbines must necessarily be run require an accurate analysis of the stresses occurring in order that these may be kept within safe limits.

The student is advised to consult the various steam-turbine catalogues and note the forms of turbine wheels.

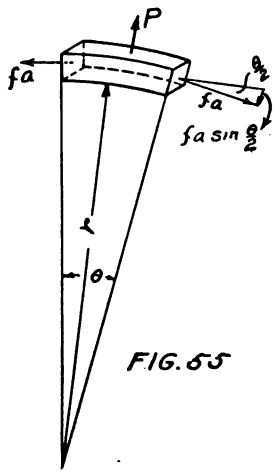

FIG. 55

As an introduction to the study of stresses in turbine wheels, let us analyze the stresses in a revolving ring. Consider an elementary portion of a ring subtended by the small angle θ. The ring is revolving such that the linear velocity of the rim is v, the angular velocity of the ring is ω, and let

a = area of section;
m = specific mass;
r = radius;
f_t = unit tension in rim;
M = mass of portion considered.

* Mr. T. J. Rodhouse of the University of Missouri kindly assisted in the work of this chapter.

The element under consideration will tend to fly out with a force

$$P=\frac{Mv^2}{r}=a\times r\theta\times m\times\frac{v^2}{r}; \quad \ldots\ldots \quad (1)$$

and there will exist at the same time a tension in the rim equal to

$$f_t a,$$

as shown in the figure. Now these forces ($f_t a$) are acting normal to the cross-section as shown, but they may be resolved into forces normal and parallel to the direction of P, and thus we may easily obtain a summation of forces.

Taking the components normal to P, we find their value is $f_t a \cos\frac{\theta}{2}$, and that they balance one another. The components parallel to P are equal to $f_t a \sin\frac{\theta}{2}$ and act in opposite direction to P, whereupon we may write

$$2fa\sin\frac{\theta}{2}=a\times r\theta\times m\times\frac{v^2}{r};$$

but since $\frac{\theta}{2}$ is taken a very small angle, $\frac{\theta}{2}=\sin\frac{\theta}{2}$, whence

$$f_t a\theta=ar\theta m\frac{v^2}{r},$$

$$f_t=mv^2. \quad \ldots\ldots\ldots \quad (2)$$

This same discussion applies to drums. Even if strengthened at the ends, if the drum is of not too small length, the above analysis may be made.

II.

Stresses in Revolving Disks.

Many turbines have their blades arranged on disk wheels. It therefore is profitable to consider the stresses occurring in such cases. Consider the stresses occurring on an elementary portion of a revolving disk, shown in the accompanying figure. These stresses are due to the centrifugal force of the elementary portion. This centrifugal force is not shown in the figure, but may be known as P. Letting the thickness of the disk be taken as unity, we find the value of P as

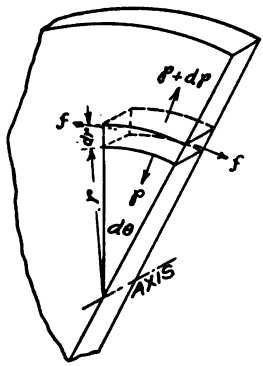

FIG. 56

$$P = \omega^2 r \cdot r d\theta \cdot dr \cdot m$$
$$= m\omega^2 r^2 d\theta dr.$$

In order that these stresses may be in equilibrium, the summation of the forces along any axis must be zero.

Take first the forces tangential and we find two equal and opposite forces, the resultants of the forces f; hence there is equilibrium in that direction.

Take now the radial components of all forces acting and their summation results as follows:

$$m\omega^2 r^2 d\theta dr = f dr d\theta + p r d\theta - (p + dp)(r + dr)d\theta.$$

Neglecting products of differentials, we have, after elimination,

$$\omega^2 r^2 dr m = f dr - p dr - r dp$$
$$= f dr - (p dr + r dp);$$

whence

$$f dr = (p dr + r dp) + \omega^2 r^2 dr m,$$

$$f = \frac{d(pr)}{dr} + m\omega^2 r^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In order that this equation may be solved it will be necessary to find another relation between f and p. This can be done in the same manner as for thick cylinders, which has just been investigated. That is, by considering the strain or elongation which the elementary portion is subjected to because of these stresses. For this let the following notation be used:

E = modulus of elasticity;
ϵ = Poisson's ratio;
λ = displacement in direction of r of any point, due to the stress.

Whereupon the strain or elongation in the direction of f at radius r is found as follows:

In Fig. 57 let AB denote the position of any fiber at distance r when there is no stress. Upon application of stress let the fiber take up a position $A'B'$ a distance $r+\lambda$ from the center. Then the original length of the fiber throughout the entire circle will be $2\pi r$ and the length when under stress will be $2\pi(r+\lambda)$. The whole fiber has been stretched or elongated a distance

$$2\pi(r+\lambda)-2\pi r,$$

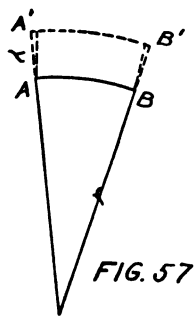

FIG. 57

from which the unit elongation or elongation per unit length is

$$\frac{2\pi(r+\lambda)-2\pi r}{2\pi r}=\frac{\lambda}{r},$$

which is the strain in the direction of f. Remembering that the strain times the modulus of elasticity is equal to the stress in the direction of the strain and that the stress in the direction of f is $f-\epsilon p$, we may write

$$E\frac{\lambda}{r}=f-\epsilon p; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and since the radial strain is $\frac{d\lambda}{dr}$, we have

$$E\frac{d\lambda}{dr}=p-\epsilon f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The solution of these equations give

$$f=\frac{E}{1-\epsilon^2}\left(\frac{\lambda}{r}+\epsilon\frac{d\lambda}{dr}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$p=\frac{E}{1-\epsilon^2}\left(\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting the results of equations (6) and (7) in equation (3), we have

$$\frac{E}{1-\epsilon^2}\left(\frac{\lambda}{r}+\epsilon\frac{d\lambda}{dr}\right)=r\frac{dp}{dr}+p\frac{dr}{dr}+m\omega^2r^2$$

$$=\frac{E}{1-\epsilon^2}\left[\frac{r}{dr}d\left(\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}\right)+\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}\right]+m\omega^2r^2;$$

from which we find

$$\frac{\lambda}{r}+\epsilon\frac{d\lambda}{dr}=\frac{r}{dr}d\left(\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}\right)+\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}+\frac{1-\epsilon^2}{E}m\omega^2r^2$$

$$=\frac{r}{dr}\left[d\left(\frac{d\lambda}{dr}\right)+\epsilon d\left(\frac{\lambda}{r}\right)\right]+\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}+\frac{1-\epsilon^2}{E}m\omega^2r^2$$

$$=\frac{r}{dr}\left[\frac{d^2\lambda}{dr^2}+\epsilon\left(\frac{rd\lambda-\lambda dr}{r^2}\right)\right]+\frac{d\lambda}{dr}+\epsilon\frac{\lambda}{r}+\frac{1-\epsilon^2}{E}m\omega^2r^2,$$

which gives on elimination

$$r\frac{d^2\lambda}{dr^2}+\frac{d\lambda}{dr}-\frac{\lambda}{r}+\frac{1-\epsilon^2}{E}m\omega^2r^2=0.$$

If this equation is now multiplied by r, it will have the form of a homogeneous linear equation; thus,

$$r^2\frac{d^2\lambda}{dr^2}+r\frac{d\lambda}{dr}-\lambda=-\frac{1-\epsilon^2}{E}m\omega^2r^3.$$

The solution of this equation contains within itself the solution of the first term equal to zero, from which the complementary function is obtained and general solution for the particular integral. The sum of the complementary function and the particular integral is the complete solution of the equation. The student is advised to look up equations of this kind in Murray's "Differential Equations" or Forsyth's "Differential Equations."

To solve this equation, let $r\frac{d}{dr}=\theta$, whereupon the equation will reduce to

$$\theta(\theta-1)\lambda-\theta\lambda-\lambda=Kr^3,$$

$$(\theta^2-1)\lambda=Kr^3,$$

where

$$K=\frac{1-\epsilon^2}{E}\omega^2m.$$

Letting $(\theta^2-1)=0$, we have as the roots of this equation

$$\theta=\pm1;$$

whereupon the complementary function is

$$Cr^{-1}+C_1r=C/r+C_1r.$$

The above equation $[(\theta^2-1)\lambda=Kr^3]$ may be written

$$\frac{\lambda}{K}=\frac{r^3}{(\theta^2-1)}.$$

Breaking the second term into partial fractions, we have

$$\frac{\lambda}{K}=\left(\frac{N_1}{\theta+1}+\frac{N_2}{\theta-1}\right)r^3,$$

and solving for N_1 and N_2, we find

$$N_1 = -\tfrac{1}{2} \quad \text{and} \quad N_2 = \tfrac{1}{2};$$

whence

$$\frac{\lambda}{K} = \left(-\frac{1}{2}\frac{1}{\theta+1} + \frac{1}{2}\frac{1}{\theta-1}\right) r^3$$

$$= \frac{1}{2}\left(\frac{1}{\theta-1} - \frac{1}{\theta+1}\right) r^3;$$

whereupon we may write the particular integral

$$= [-\tfrac{1}{2} r^{-1} \int r^{-1-1} r^3 dr + \tfrac{1}{2} r^{-1} \int r^{+1-1} r^3 dr] m\omega^2 \frac{1-\varepsilon^2}{E}$$

$$= [-\tfrac{1}{4} r^3 + \tfrac{1}{8} r^3] m\omega^2 \frac{1-\varepsilon^2}{E}$$

$$= -\tfrac{1}{8} r^3 m\omega^2 \left(\frac{1-\varepsilon^2}{E}\right).$$

The complete solution being the sum of the complementary function and the particular integral gives

$$\lambda = \frac{C}{r} + C_1 r - \tfrac{1}{8} r^3 m\omega^2 \left(\frac{1-\varepsilon^2}{E}\right), \quad \ldots \quad (9)$$

$$\frac{d\lambda}{dr} = -\frac{C}{r^2} + C_1 - \tfrac{3}{8} r^3 m\omega^2 \left(\frac{1-\varepsilon^2}{E}\right), \quad \ldots \quad (10)$$

where C and C_1 are constants depending on whether the disk is solid or has a hole in its center.

Let the values of C and C_1 be found as follows:

1st. For solid disk:

$$\lambda = 0 \quad \text{when} \quad r = 0,$$

whence, from equation (9),

$$\lambda r = C + C_1 r^2 - \frac{(1-\varepsilon^2) m\omega^2 r^4}{8E},$$

$$\dot{o} = C + o - o,$$

$$C = o;$$

$$\therefore \frac{\lambda}{r} = o + C_1 - \frac{(1-\varepsilon^2) m\omega^2 r^2}{8E};$$

whence

$$\lambda = C_1 r - \frac{(1-\varepsilon^2) m\omega^2 r^3}{8E},$$

and

$$\frac{d\lambda}{dr} = -o + C_1 - \frac{3(1-\varepsilon^2) m\omega^2 r^2}{8E}.$$

Substituting these values in equation (7), we have

$$p = \frac{E}{1-\varepsilon^2}\left[\varepsilon\left(C_1 - \frac{(1-\varepsilon^2) m\omega^2 r^2}{8E}\right) + C_1 - \frac{3(1-\varepsilon^2) m\omega^2 r^2}{8E}\right];$$

but $p = o$ when $r = r_1 =$ outer radius, whence

$$o = \frac{E}{1-\varepsilon^2}\left[C_1(1+\varepsilon) - \frac{(3+\varepsilon)(1-\varepsilon^2) m\omega^2 r_1^2}{8E}\right],$$

$$C_1 = \frac{(1-\varepsilon)(3+\varepsilon) m\omega^2 r_1^2}{8E}.$$

Using this value of C_1 in the equations (9) and (10) and substituting as solved in equation (7), we find

$$p = \frac{\omega^2 m}{8}(3+\varepsilon)(r_1^2 - r^2), \quad \ldots\ldots \quad (11)$$

which we see is a maximum for $r=0$, whence

$$p_{\max.}=\tfrac{1}{8}(\omega^2 m r_1^2)(3+\epsilon),$$

which is the value of the stress at the center of the disk. Solving for f in equation (6), we find

$$f=\frac{m\omega^2}{8}[(3+\epsilon)r_1^2-(1+3\epsilon)r^2], \quad . \quad . \quad . \quad . \quad (12)$$

which is also a maximum for $r=0$, whence

$$f_{\max.}=\tfrac{1}{8}(\omega^2 m r_1^2)(3+\epsilon)=p_{\max.}. \quad . \quad . \quad . \quad . \quad (12')$$

2d. Disk with hole in center:

Let r_0 = radius of hole.

Then $p=0$ when $r=r_0$ and $r=r_1$.

Substituting the values of $\frac{\lambda}{r}$ and $\frac{d\lambda}{dr}$ as found in equations (9) and (10) in equations (6) and (7), we have

$$f=\frac{E}{1-\epsilon^2}\left[\frac{C}{r^2}(1-\epsilon)+C_1(1+\epsilon)-\frac{(1+3\epsilon)(1-\epsilon^2)m\omega^2 r^2}{8E}\right] \quad (13)$$

and

$$p=\frac{E}{1-\epsilon^2}\left[\frac{C}{r^2}(\epsilon-1)+C_1(1+\epsilon)-\frac{(3+\epsilon)(1-\epsilon^2)m\omega^2 r^2}{8E}\right], \quad (14)$$

whence

$$f=E\left[\frac{C}{r^2(1+\varepsilon)}+\frac{C_1}{1-\varepsilon}-\frac{(1+3\varepsilon)m\omega^2r^2}{8E}\right], \quad . \quad . \quad (15)$$

$$p=E\left[-\frac{C}{r^2(1+\varepsilon)}+\frac{C_1}{1-\varepsilon}-\frac{(3+\varepsilon)m\omega^2r^2}{8E}\right]. \quad . \quad . \quad (16)$$

When $r=r_1$,

$$p=0=E\left[-\frac{C}{r_1^2(1+\varepsilon)}+\frac{C_1}{1-\varepsilon}-\frac{(3+\varepsilon)m\omega^2r_1^2}{8E}\right]. \quad . \quad (17)$$

When $r=r_0$,

$$p=0=E\left[-\frac{C}{r_0^2(1+\varepsilon)}+\frac{C_1}{1-\varepsilon}-\frac{(3+\varepsilon)m\omega^2r_0^2}{8E}\right]. \quad . \quad (18)$$

From (17),

$$\frac{C}{1+\varepsilon}=\frac{C_1r_1^2}{1-\varepsilon}-\frac{(3+\varepsilon)m\omega^2r_1^4}{8E}.$$

From (18),

$$\frac{C}{1+\varepsilon}=\frac{C_1r_0^2}{1-\varepsilon}-\frac{(3+\varepsilon)m\omega^2r_0^4}{8E};$$

whence

$$C_1=\frac{(1+\varepsilon)(3+\varepsilon)m\omega^2(r_1^2+r_0^2)}{8E},$$

and substituting C_1 in (17), we find

$$C=\frac{(1+\varepsilon)(3+\varepsilon)m\omega^2r_1^2r_0^2}{8E}.$$

Substituting these values in (15) and (16), we find

$$f=\frac{\omega^2 m}{8}\left[(\varepsilon+3)\left(\frac{r_1^2 r_0^2}{r^2}+r_1^2+r_0^2\right)-(3\varepsilon+1)r^2\right], \quad . \quad (19)$$

$$p=\frac{\omega^2 m}{8}\left[(\varepsilon+3)\left(r_1^2+r_0^2-\frac{r_1^2 r_0^2}{r}-r^2\right)\right]. \quad . \quad . \quad . \quad . \quad (20)$$

It is evident when $r=r_0$ that f is a maximum, that is, at the circumference of the hole. Making this substitution, we have

$$f_{\text{max.}}=\frac{m\omega^2}{4}[(3+\varepsilon)r_1^2+(1-\varepsilon)r_0^2], \quad . \quad . \quad . \quad (21)$$

and also

$$p=0.$$

If the hole is so small that r_0^2 may be neglected, it will appear that

$$f=\frac{m\omega^2}{4}[(3+\varepsilon)r_1^2],$$

which it will be seen is just twice the stress occurring at the center of a disk having no hole in the center (eq. 12′).

NOTE.—The student must not think that substituting $r_0=0$, in equation (21), will give the stress at the center of a disk having no hole. Such a substitution is incorrect here, but may be made in equation (19) if desired, and the correct result obtained. Why this is so I think is apparent.

It will be noted that the De Laval turbine wheels are made solid throughout for the reason given above.

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